BM1T, 1E, 1.12.2016

① a) $$\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\underline{\underline{\frac{x+y}{xy}}}$$

b) $$\frac{1}{a-b}-\frac{1}{b-a}=\frac{1}{a-b}-\frac{1}{-(a-b)}=$$
$$=\frac{1}{a-b}+\frac{1}{a-b}=\underline{\underline{\frac{2}{a-b}}}$$

c) $$1-\frac{x}{x+1}=\frac{x+1}{x+1}-\frac{x}{x+1}$$
$$=\frac{x+1-x}{x+1}=\underline{\underline{\frac{1}{x+1}}}$$

d) $$\frac{2a}{a^2-9}-\frac{3}{2a-6}=\frac{2a}{(a+3)(a-3)}-\frac{3}{2(a-3)}$$
$$\cancel{=\frac{2\cdot 2a}{2(a+3)(a-3)}-\frac{3(a+b)}{2(a-3)(a+3)}}$$
$$\cancel{=\frac{4a-[3(a+b)]}{2(a+3)(a-3)}=\frac{4a-[3a+3b]}{''}}$$
$$\cancel{=\frac{4a-3a-3b}{''}=\frac{a-3b}{2(a+3)(a-3)}}$$

$$=\frac{2\cdot 2a}{2(a+3)(a-3)}-\frac{3(a+3)}{2(a+3)(a-3)}$$
$$=\frac{4a-(3a+9)}{''}=\frac{4a-3a-9}{''}=\frac{a-9}{''}$$
$$=\underline{\underline{\frac{a-9}{2(a+3)(a-3)}}}$$

(2) a) $\frac{1}{x^2+2x+1} - \frac{1}{x^2-1} = \frac{1}{(x+1)^2} - \frac{1}{(x+1)(x-1)}$

$= \frac{1(x-1)}{(x-1)(x+1)^2} - \frac{1(x+1)}{(x-1)(x+1)^2}$

$= \frac{x-1-(x+1)}{(x+1)^2(x-1)} = \frac{x-1-x-1}{(x+1)^2(x-1)}$

$= -\frac{2}{(x+1)^2(x-1)}$

b) $\frac{a+2b}{a^2+ab} + \frac{1}{a+b} - \frac{1}{a} = \frac{a+2b}{a(a+b)} + \frac{1}{a+b} - \frac{1}{a}$

$= \frac{a+2b}{a(a+b)} + \frac{a}{a(a+b)} - \frac{a+b}{a(a+b}$

$= \frac{a+2b+a-(a+b)}{''} = \frac{a+2b+a-a-b}{''}$

$= \frac{a+b}{a(a+b)} = \frac{1}{a}$

2c

$$\frac{x^2+2x-3}{x^2-2x+1} - \frac{3x+3}{x^2-1} = \frac{(x-1)(x+3)}{(x-1)^2} - \frac{3(x+1)}{(x+1)(x-1)}$$

$$= \frac{(x-1)(x+3)(x+1)}{(x-1)^2(x+1)} - \frac{3(x+1)(x-1)}{(x-1)^2(x+1)}$$

$$= \frac{(x-1)(x+3)(x+1) - [3(x+1)(x-1)]}{(x-1)^2(x+1)}$$

Zwischenrechnung: $(x-1)(x+1)(x+3) = (x^2-1)(x+3)$

$$= x^3+3x^2-x-3$$

$$3(x+1)(x-1) = 3x^2-3$$

$$\underline{x^3}+\underline{3x^2}-x-3-\underline{3x^2}+3 = x^3-x$$

$$= x(x^2-1) = x(x+1)(x-1)$$

$$= \frac{x\cancel{(x+1)}\cancel{(x-1)}}{(x-1)^2\cancel{(x+1)}} = \underline{\underline{\frac{x}{x-1}}}$$

2d

$$\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2-1} - 1 = \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{(x+1)(x-1)} - 1$$

$$= \frac{(x+1)}{(x+1)(x-1)} - \frac{(x-1)}{(x+1)(x-1)} - \frac{2}{(x+1)(x-1)} - \frac{(x+1)(x-1)}{(x+1)(x-1)}$$

$$= \frac{x+1-(x-1)-2-((x+1)(x-1))}{\text{''}}$$

$$= \frac{x+1-x+1-2-x^2+1}{\text{''}} = \frac{-x^2+1}{(x+1)(x-1)}$$

$$= \frac{-(x^2-1)}{(x+1)(x-1)} = -\frac{(x+1)(x-1)}{(x+1)(x-1)} = \underline{\underline{-1}}$$

③

a) $$\frac{\frac{1}{x+1}-\frac{1}{x-1}}{\frac{1}{x^2-1}} = \frac{\frac{(x-1)}{(x+1)(x-1)}-\frac{(x+1)}{(x+1)(x-1)}}{\frac{1}{(x+1)(x-1)}}$$

$$= \frac{x-1-(x+1)}{u} = \frac{x-1-x-1}{u} = \frac{-2}{u}$$

$$= \frac{-\frac{2}{(x+1)(x-1)}}{\frac{1}{(x+1)(x-1)}} = \cancel{-\frac{2}{}}$$

$$= -\frac{2}{(x+1)(x-1)} \cdot \frac{(x+1)(x-1)}{1} = \underline{\underline{-2}}$$

b) $$\frac{\frac{1}{a}-a^3}{a-\frac{1}{a^3}} = \frac{\frac{1-a^4}{a}}{\frac{a^4-1}{a^3}} = \frac{\frac{(1-a^2)(1+a^2)}{a}}{\frac{(a^2+1)(a^2-1)}{a^3}}$$

$$= -\frac{\cancel{(a^2-1)}\cancel{(a^2+1)}}{a} \cdot \frac{a^3}{\cancel{(a^2+1)}\cancel{(a+1)(a-1)}}$$

$$= \underline{\underline{-a^2}}$$

4

$$\frac{\frac{q^2-p^2}{r}}{rq^2-rp^2-\frac{q^2rp-p^3r}{q}} = \frac{\frac{(q+p)(q-p)}{r}}{\frac{rq^3-rp^2q-q^2rp-p^3r}{q}}$$

$$rq^3-rp^2q-q^2rp-p^3r$$
$$= rq(q^2-p^2)-rp(q^2-p^2)$$
$$= (rq+rp)(q^2-p^2) = r(q+p)(q+p)(q-p)$$

$$= \frac{(\cancel{q+p})(q-p)}{r} \cdot \frac{q}{r(\cancel{q+p})(q+p)(q-p)}$$

$$= \underline{\underline{\frac{q}{r^2(q-p)}}}$$