TBM 1A, 2. 10. 2015

Algebra I

(1) a) $x - [y - (z - x) + \{-z + (-x)\}]$

$= x - [y - z + x + \{-z - x\}]$

$= x - [y - z + x - z - x]$

$= x - [y - 2z] = \underline{\underline{x - y + 2z}}$

b) $-[c - b - \{b - (-2 + 4c)\} + \{-2c - (3c - 2b) + 2\}]$

$= -[c - b - \{b + 2 - 4c\} + \{-2c - 3c + 2b + 2\}]$

$= -[c - b - \{b + 2 - 4c\} + \{-5c + 2b + 2\}]$

$= -[c - b - b - 2 + 4c - 5c + 2b + 2]$

$= -[0] = \underline{\underline{0}}$

(2) a) $-(-5)^2 = -(-5)(-5) = \underline{\underline{-5^2}}$

b) $-(-x)^3 = -(-x)(-x)(-x) = \underline{\underline{x^3}}$

c) $-a^2 \cdot (-a)^2 = -a^2 \cdot a^2 = \underline{\underline{-a^4}}$

d) $-(-n)^{77} = -(-n^{77}) = \underline{\underline{n^{77}}}$

(3) $(xy^2 + 3z)^2 = (xy^2)^2 + 2xy^2 \cdot 3z + (3z)^2$

$= \underline{\underline{x^2y^4 + 6xy^2z + 9z^2}}$

(4)

a) $3xt^2 + 9tx + 6x$

$= 3x(t^2 + 3t + 2)$

$= \underline{3x(t+1)(t+2)}$

b) $a^2 - 3a - 10$

$= \underline{(a+2)(a-5)}$

c) $3x^3y - 3x^2y - 60xy$

$= 3xy(x^2 - x - 20)$

$= \underline{3xy(x+4)(x-5)}$

d) $2x^2 + 6xy + 5x + 15y$

$= 2x(x+3y) + 5(x+3y)$

$= \underline{(2x+5)(x+3y)}$

e) $\frac{1}{4}x^2 - \frac{1}{9}y^2$

$= \underline{\left(\frac{1}{2}x + \frac{1}{3}y\right)\left(\frac{1}{2}x - \frac{1}{3}y\right)}$

f) $a^4 + 2a^2b^2 + b^4$

$= \underline{(a^2 + b^2)^2}$

(5)

$$\frac{\frac{5x^2-5}{x-1}}{\frac{5x^2+10x+5}{2x+2}} = \frac{\frac{5(x^2-1)}{x-1}}{\frac{5(x^2+2x+1)}{2(x+1)}}$$

$$= \frac{\frac{5(x+1)(x-1)}{(x-1)}}{\frac{5(x+1)^2}{2(x+1)}} = \frac{5(x+1)}{\frac{5}{2}(x+1)}$$

$$= \frac{5}{\frac{5}{2}} = \frac{5}{1} \cdot \frac{2}{5} = \underline{\underline{2}}$$

(6) $(x+12)(x-3) = 54$

$x^2 + 12x - 3x - 36 = 54 \quad / -54$

$x^2 + 9x - 90 = 0$

$x^2 + 9x - 90 = 0 \qquad 90 = 6 \cdot 15$

$(x-6)(x+15) = 0$

$x_1 = 6 \; ; \; x_2 = -15$

Test: $(6+12)(6-3) = 3 \cdot 18 = 54$

$(-15+12)(-15-3) = (-3)(-18) = 54$

(7)

a) $\frac{x^2-y^2}{(y-x)^2} = \frac{(x+y)(x-y)}{(x-y)^2}$

$x-y \neq y-x$, aber: $(x-y)^2 = (y-x)^2$

$= \frac{(x+y)(x-y)}{(x-y)(x-y)} = \underline{\underline{\frac{x+y}{x-y}}}$

b) ~~$x \neq 0 \neq 1$~~

$x^4-1 = (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$

$x^4+x^3+x^2+x = x^3(x+1) + x(x+1)$

$= (x^3+x)(x+1)$

$= x(x^2+1)(x+1)$

$\hookrightarrow \frac{(x^2+1)(x-1)(x-1)}{x(x^2+1)(x+1)} = \underline{\underline{\frac{x-1}{x}}}$