TBM 1B, 4.12.2014

① a) $\frac{(r-s)^2}{s-r} = \frac{(s-r)^2}{(s-r)} = s-r$

b) $\frac{a^3-ab^2}{a^2x-abx-2a^2y+2aby} = \frac{a(a^2-b^2)}{a(ax-bx-2ay+2by)}$

$= \frac{(a+b)(a-b)}{x(a-b)-2y(a-b)} = \frac{(a+b)(a-b)}{(a-b)(x-2y)}$

$= \frac{a+b}{x-2y}$

c) $\frac{2a^2-6a-20}{3a^2-18a+15} = \frac{2(a^2-3a-10)}{3(a^2-6a+5)}$

$= \frac{2(a+2)(a-5)}{3(a-1)(a-5)} = \frac{2(a+2)}{3(a-1)}$

d) $\frac{a^4-16b^4}{a^3-2a^2b+4ab^2-8b^3} = \frac{(a^2+4b^2)(a^2-4b^2)}{a^2(a-2b)+4b^2(a-2b)}$

$= \frac{(a^2+4b^2)(a+2b)(a-2b)}{(a^2+4b^2)(a-2b)} = a+2b$

(2) a) $\frac{1}{a-b} - \frac{1}{b-a} = \frac{1}{a-b} - \frac{1}{-(a-b)}$

$= \frac{1}{a-b} + \frac{1}{a-b} = \underline{\underline{\frac{2}{a-b}}}$

b) $1 - \frac{x}{x+1} = \frac{x+1}{x+1} - \frac{x}{x+1} = \frac{x+1-x}{x+1} = \underline{\underline{\frac{1}{x+1}}}$

c) $\frac{x+1}{x} - \frac{x+2}{x+1} = \frac{(x+1)(x+1)}{x(x+1)} - \frac{x(x+2)}{x(x+1)}$

$= \frac{x^2+2x+1-(x^2+2x)}{x(x+1)} = \frac{x^2+2x+1-x^2-2x}{x(x+1)}$

$= \underline{\underline{\frac{1}{x(x+1)}}}$

d) $\frac{2}{x-5} - \frac{8}{x^2-x-20} = \frac{2}{x-5} - \frac{8}{(x+4)(x-5)}$

$= \frac{2(x+4)}{(x+4)(x-5)} - \frac{8}{(x+4)(x-5)} = \frac{2x+8-8}{(\quad)(\quad)}$

$= \underline{\underline{\frac{2x}{(x+4)(x-5)}}}$

e) $\frac{2a}{a^2-9} - \frac{3}{2a-6} = \frac{2a}{(a+3)(a-3)} - \frac{3}{2(a-3)}$

$= \frac{2 \cdot 2a}{2(a+3)(a-3)} - \frac{3(a+3)}{2(a-3)(a+3)}$

$= \frac{4a-(3a+9)}{2(a+3)(a-3)} = \frac{4a-3a-9}{\quad\quad}$

$= \underline{\underline{\frac{a-9}{2(a-3)(a+3)}}}$

f) $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2-1} - 1$

$= \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{(x+1)(x-1)} - 1$ $\quad |\cdot (x+1)(x-1)$

$= \frac{x+1}{(x+1)(x-1)} - \frac{x-1}{(x+1)(x-1)} - \frac{2}{(x+1)(x-1)} - \frac{(x+1)(x-1)}{(x+1)(x-1)}$

$= \frac{x+1-(x-1)-2-((x+1)(x-1))}{(x+1)(x-1)}$

$= \frac{x+1-x+1-2-(x^2-1)}{(x+1)(x-1)}$

$= \frac{x+1-x+1-2-x^2+1}{(x+1)(x-1)}$

$= \frac{-x^2+1}{(x+1)(x-1)} = \frac{1-x^2}{(x+1)(x-1)} =$

$\frac{(y+x)(y-x)}{(x+1)(x-1)} = \frac{-(x+y)(x-y)}{}$

$\frac{-(x+1)(x-1)}{(x+1)(x-1)} = \underline{\underline{-1}}$

(3) a) $$\frac{\frac{x-y}{b-a}}{\frac{y-x}{a-b}} = \frac{x-y}{b-a} \cdot \frac{a-b}{y-x}$$

$$= -\frac{x-y}{a-b} \cdot \left(-\frac{a-b}{x-y}\right)$$

$$= \frac{x-y}{a-b} \cdot \frac{a-b}{x-y} = \underline{\underline{1}}$$

b) $$\frac{\frac{ab-a^2}{3}}{a-\frac{a^2}{b}} = \frac{\frac{a(b-a)}{3}}{\frac{ab-a^2}{b}} = \frac{a(b-a)}{3} \cdot \frac{b}{a(b-a)}$$

$$= \underline{\underline{\frac{b}{3}}}$$

c) $$\frac{\frac{m}{m+1} - \frac{m}{m+2}}{\frac{m}{m+2} - \frac{2m}{m+1}} = \frac{\frac{m(m+2) - m(m+1)}{(m+1)(m+2)}}{\frac{m(m+1) - 2m(m+2)}{(m+1)(m+2)}}$$

$$= \frac{m^2+2m-m^2-m}{(m+1)(m+2)} \cdot \frac{(m+1)(m+2)}{m^2+m-2m^2-4m}$$

$$= \frac{m}{-m^2-3m} = \frac{m}{-m(m+3)} = \underline{\underline{-\frac{1}{m+3}}}$$

(4)

$$\frac{1-\frac{1}{x}}{1-\frac{1-\frac{1}{x}}{2-\frac{1}{x}}} = \frac{\frac{x-1}{x}}{1-\frac{\frac{x-1}{x}}{\frac{2x-1}{x}}} \quad \Big\} (*)$$

*) $$\frac{\frac{x-1}{x}}{\frac{2x-1}{x}} = \frac{x-1}{x} \cdot \frac{x}{2x-1} = \frac{x-1}{2x-1}$$

$$\frac{\frac{x-1}{x}}{1-\frac{x-1}{2x-1}} = \frac{\frac{x-1}{x}}{\frac{2x-1-(x-1)}{2x-1}} = \frac{\frac{x-1}{x}}{\frac{x}{2x-1}}$$

$$= \frac{x-1}{x} \cdot \frac{2x-1}{x} = \frac{(x-1)(2x-1)}{x^2}$$