TBM 1E, 23.1.14

(1) a) $\sqrt[3]{8^2} = \left((2^3)^2\right)^{1/3} = 2^{3\cdot 2\cdot \frac{1}{3}} = 2^2 = \underline{4}$

b) $\sqrt[6]{(0.000001)^2} = \left((10^{-6})^2\right)^{1/6} = 10^{-2} = \underline{0.01}$

(2) a) $10^{3/5} = \underline{\sqrt[5]{10^3}}$

b) $(xy)^{\frac{n+1}{n}} = \underline{\sqrt[n]{(xy)^{n+1}}}$

c) $s^{-2/3} = \frac{1}{s^{2/3}} = \underline{\frac{1}{\sqrt[3]{s^2}}}$

d) $\frac{c^{-2/3}}{a\cdot b^{-5}} = \frac{b^5}{a\,c^{2/3}} = \underline{\frac{b^5}{a\sqrt[3]{c^2}}}$

(3) a) $x^{\frac{1}{2}}\cdot x^{\frac{1}{2}} = \left(x^{1/2}\right)^2 = \underline{x}$

b) $a^{-1/2}\cdot a^{1/3} = a^{-\frac{1}{2}+\frac{1}{3}} = \underline{a^{-\frac{1}{6}}}$

c) $\frac{c^{5/6}}{c^{1/2}} = c^{\frac{5}{6}-\frac{1}{2}} = \underline{c^{\frac{1}{3}}}$

d) $\frac{s^{1/2}}{s^{-1/2}} = s^{\frac{1}{2}-\left(-\frac{1}{2}\right)} = s^{\frac{1}{2}+\frac{1}{2}} = s^1 = \underline{s}$

(4) a) $\sqrt{\frac{y}{x}}\sqrt{xy} = \sqrt{\frac{y\cdot xy}{x}} = \sqrt{y^2} = \underline{y}$

b) $\sqrt{x^3}\sqrt[3]{x^2} = x^{\frac{3}{2}}\cdot x^{\frac{2}{3}} = x^{\frac{9}{6}+\frac{4}{6}} = \underline{x^{13/6}}$

c) $\sqrt{a^2b^4}\sqrt[3]{a^4b^2} = ab^2\cdot a^{4/3}\cdot b^{2/3}$

$= a^{7/3}b^{8/3}$ oder $\left(a^7b^8\right)^{1/3}$

d) $\sqrt[n]{a^{n+5}}\sqrt[n]{a^{n-5}} = \sqrt[n]{a^{n+5+n-5}}$

$= \sqrt[n]{a^{2n}} = \left(a^{2n}\right)^{1/n} = \underline{a^2}$

(5) a) $\left(a^{\frac{1}{2}}\right)^{\frac{3}{4}} = a^{3/8} = \sqrt[8]{a^3}$

b) $\left(27x^3\right)^{-1/3} = 27^{-1/3} \cdot x^{3\cdot(-1/3)}$

$= \frac{1}{27^{1/3}} \cdot x^{-1} = \frac{1}{\sqrt[3]{27}} \cdot \frac{1}{x} = \underline{\underline{\frac{1}{3x}}}$

c) $\left(a^{\frac{1}{2}} b^{\frac{1}{3}}\right)^{3/4} = a^{3/8} b^{3/12} = a^{3/8} b^{1/4} = \sqrt[8]{a^3}\sqrt[4]{b}$

oder $\underline{\underline{\sqrt[8]{a^3 b^2}}}$

d) $\left(\frac{x^{\frac{1}{2}} y^{-\frac{1}{3}}}{z^{-2}}\right)^2 \quad \frac{x^1 \cdot z^4}{y^{\frac{2}{3}}} = \underline{\underline{\frac{x z^4}{\sqrt[3]{y^2}}}}$

(6) $\sqrt{a\sqrt{a\sqrt{a}}} = \left(a\left(a \cdot a^{\frac{1}{2}}\right)^{1/2}\right)^{1/2}$

$= a^{1/2} \cdot \left(a \cdot a^{\frac{1}{2}}\right)^{1/4} = a^{\frac{1}{2}} \left(a^{\frac{3}{2}}\right)^{\frac{1}{4}} = a^{\frac{1}{2}} a^{\frac{3}{8}}$

$= \underline{\underline{a^{7/8}}} = \sqrt[8]{a^7}$

b) $\sqrt[6]{\sqrt[5]{\sqrt[5]{x^{155}}}} = \left(\left(x^{\frac{155}{5}}\right)^{\frac{1}{5}}\right)^{\frac{1}{6}}$

$= x^{\frac{15\cdot 5}{55} \cdot \frac{1}{6}} = \underline{\underline{x^{\frac{1}{2}}}} = \sqrt{x}$