ZI4D3, 11.4.2018:
Logarithmus I

(1) a) $\log(10'000) = \log(10^4) = \underline{\underline{4}}$

b) $\log(0.000'01) = \log(10^{-5}) = \underline{\underline{-5}}$

c) $\log\left(\sqrt[5]{100'000^2}\right) = \log\left(\left((10^5)^2\right)^{1/5}\right) = \log(10^2) = \underline{\underline{2}}$

d) $\log\left(\sqrt[7]{1'000^2}\right) = \log\left(\left((10^3)^2\right)^{1/7}\right) = \log(10^{6/7}) = \underline{\underline{\frac{6}{7}}}$

e) $\log_5(25) = \log_5(5^2) = \underline{\underline{2}}$

f) $\log_3\left(\left(\frac{1}{9}\right)^2\right) = \log_3\left(\left(\frac{1}{3^2}\right)^2\right) = \log_3\left((3^{-2})^2\right) = \log(3^{-4}) = \underline{\underline{-4}}$

g) $\log_2(8^5) = \log_2\left((2^3)^5\right) = \log_2(2^{15}) = \underline{\underline{15}}$

h) $\log_2\left(\sqrt[3]{\frac{1}{4}}\right) = \log_2\left(\sqrt[3]{\frac{1}{2^2}}\right) = \log_2\left((2^{-2})^{1/3}\right) = \underline{\underline{-\frac{2}{3}}}$

(2) a) $\log_2(10) = y \iff 2^y = 10 \quad | \log$

$y \cdot \log 2 = \log 10 = 1$

$y = \log_2(10) = \frac{1}{\log(2)}$

b) $\log_{13}(2197) = y \iff 13^y = 2197 \quad | \log$

$y \cdot \log(13) = \log(2197)$

$y = \log_{13}(2197) = \frac{\log(2197)}{\log(13)}$

(3)

a) $$Log\left(\frac{a^{-1}b^{2}}{c^{-3}d^{4}}\right) = Log\left(a^{-1}b^{2}c^{3}d^{-4}\right)$$
$$= log(a^{-1}) + log(b^{2}) + log(c^{3}) + log(d^{-4})$$
$$= -Log(a) + 2Log(b) + 3Log(c) - 4Log(d)$$

b) $$Log\left(\left(\frac{x^{3}y^{-5}}{c^{7}}\right)^{-2}\right) = Log\left(\frac{x^{-6}y^{10}}{c^{-14}}\right)$$
$$= Log\left(x^{-6}y^{10}c^{14}\right)$$
$$= -6Log(x) + 10log(y) + 14log(c)$$

(4)

a) $$2log(x) - 3log(y) - 4log(z)$$
$$= log(x^{2}) + log(y^{-3}) + log(z^{-4})$$
$$= Log(x^{2}y^{-3}z^{-4}) = Log\left(\frac{x^{2}}{y^{3}z^{4}}\right)$$

b) $$\frac{1}{2}Log(a) + \frac{1}{2}log\, b - 2log(c) - 2log(d)$$
$$= Log(a^{1/2}) + log(b^{1/2}) + log(c^{-2}) + log(d^{-2})$$
$$= log\left(a^{1/2}b^{1/2}c^{-2}d^{-2}\right)$$
$$= Log\left(\frac{(ab)^{1/2}}{(cd)^{2}}\right)$$