6HD3, 24.5.2018

(1.) $5 \cdot 10^x = 485 \quad /:5$

$10^x = 97 \quad / \log$

$x \cdot \log(10) = \log(97) \qquad \log(10) = 1$

$x = \log(97)$

$x \approx 1.987 \qquad (1.986'771...)$

(2.) $33 \cdot 10^{x-5} = 10^6 \quad /:33$

$10^{x-5} = \frac{10^6}{33} \quad / \log$

$(x-5) \cdot \log(10) = \log\left(\frac{10^6}{33}\right) \qquad \log(10) = 1$

$x - 5 = \quad "$

$x = \log\left(\frac{10^6}{33}\right) + 5$

$x \approx 9.481 \qquad (9.481'486...)$

(3) $343 \cdot 7^{x-10} = 512 \cdot 8^{x-10} \quad / : 343 : 8^{x-10}$

$$\frac{7^{x-10}}{8^{x-10}} = \frac{512}{343}$$

$$\left(\frac{7}{8}\right)^{x-10} = \frac{512}{343} \quad / \log$$

$$(x-10) \cdot \log\left(\tfrac{7}{8}\right) = \log\left(\tfrac{512}{343}\right) \quad / : \log \tfrac{7}{8}$$

$$x - 10 = \frac{\log\left(\tfrac{512}{343}\right)}{\log\left(\tfrac{7}{8}\right)} \quad / + 10$$

$$x = \frac{\log\left(\tfrac{512}{343}\right)}{\log\left(\tfrac{7}{8}\right)} + 10$$

$$x \overset{\text{TR}}{=} -3 + 10 = 7$$

oder: $512 = 8^3$, $343 = 7^3$

$$\left(\frac{7}{8}\right)^{x-10} = \left(\frac{8}{7}\right)^3 = \left(\frac{7}{8}\right)^{-3}$$

$$\hookrightarrow \quad x - 10 = -3$$

$$x = 7$$

(4) $\log(x+0.01) = -2$

Als Kopfrechnung:

$\log(\square) = -2 \Rightarrow \square = 10^{-2} = 0.01$

$\Rightarrow x + 0.01 = 0.01$

$\underline{\underline{x = 0}}$

$\log(x+0.01) = -2 \iff 10^{-2} = x + 0.01$

$\underline{\underline{x = 0}}$

(5) $\frac{1}{4}\log_3(5x+1) = 1 \quad | \cdot 4$

$\log_3(5x+1) = 4$

$\Rightarrow 5x + 1 = 3^4 = 81 \quad | -1$

$5x = 80 \quad | : 5$

$\underline{\underline{x = 16}}$

(6) $K_0 = 17'500.-$; $p = 1.25\%$ p.a.

$K_n = K_0 + 4'114.95 = 21'614.95$

$21'614.95 = 17'500 \cdot (1 + 0.0125)^n$

$\frac{21'614.95}{17'500} = 1.0125^n \quad / \log$

$\log(\ ^n\ ) = n \cdot \log(1.0125)$

$n = \frac{\log\left(\frac{21614.95}{17'500}\right)}{\log(1.0125)} = 17.0001...$

17 Jahre

(7) K_0 = Neuwert; Abnahme um 75%, bedeutet, dass der Restwert 25% des Neuwertes beträgt. $p = -10\% = -0.1$

$K_n = K_0 (1 - 10\%)^n \quad K_n = 0.25\, K_0$

$\frac{1}{4} \cdot K_0 = K_0 (0.9)^n \quad / : K_0$

$\frac{1}{4} = (0.9)^n \quad / \log$

$\log(1/4) = n \log(0.9)$

$n = \frac{\log(0.25)}{\log(0.9)} = 13.157....$

Zwischen dem 13. und 14. Jahr

(8) Lara: $K_0 = 2'000.-$, $p = +25\%$

Carlo: $K_0 = 1'000'000.-$ $p = -25\%$

$2000(1+25\%)^n = 1'000'000(1-25\%)^n \quad /:2000$

$1.25^n = 500 \cdot 0.75^n$

($1.25 = 5/4$, $0.75 = 3/4$)

$\left(\frac{5}{4}\right)^n = 500\left(\frac{3}{4}\right)^n$

$\left(\frac{\frac{5}{4}}{\frac{3}{4}}\right)^n = 500$

$\left(\frac{5}{3}\right)^n = 500 \quad /\log$

$n \cdot \log\left(\frac{5}{3}\right) = \log(500)$

$n = \frac{\log(500)}{\log(5/3)} = 12.165'811...$

↳ zwischen dem 12. und dem 13. Jahr.

↳ 30'200.85 Franken