

GestBM 6M

Logarithmen

①

a) 8

b) $\log_3 243 = \log_3 3^5 = 5$

c) $\log 10^{-12} = -12$

d) $\log_2 (8^{10}) = \log_2 ((2^3)^{10}) = \log_2 2^{30} = 30$

e) -19

f) $\log_3 27^5 = \log_3 (3^3)^5 = \log_3 3^{15} = 15$

g) -12

h) $\log_2 1624^{13} = \log_2 (2^{10})^{13} = \log_2 2^{130} = \underline{\underline{130}}$

i) $\log_2 0.125 = \log_2 \frac{1}{8} = \log_2 2^{-3} = -3$

k) $\log_5 0.2 = \log_5 \frac{1}{5} = \log_5 5^{-1} = -1$

l) $\log_3 \left(\frac{1}{9}\right)^2 = \log_3 (9^{-1})^2 = \log_3 (3^{-2})^2 = -4$

↳ $\log_3 \left(\frac{1}{3^2}\right)^2 = \log_3 \left(\frac{1}{3}\right)^4 = \log_3 3^{-4} = -4$

m) $\log_{15} \frac{1}{225} = \log_{15} \frac{1}{15^2} = \log_{15} 15^{-2} = -2$

$$n) \log \sqrt{10} = \log 10^{\frac{1}{2}} = \frac{1}{2}$$

$$o) \log \sqrt{1000} = \log (10^3)^{\frac{1}{2}} = \log 10^{\frac{3}{2}} = \frac{3}{2}$$

$$p) \log \sqrt[3]{100} = \log (10^2)^{\frac{1}{3}} = \log 10^{\frac{2}{3}} = \frac{2}{3}$$

$$q) \log \sqrt[5]{10} = \log 10^{\frac{1}{5}} = \frac{1}{5}$$

$$r) \log_3 \sqrt{\frac{1}{27}} = \log_3 \left(\frac{1}{3^3} \right)^{\frac{1}{2}} = \log_3 (3^{-3})^{\frac{1}{2}} = \log_3 3^{-\frac{3}{2}} = \underline{\underline{-\frac{3}{2}}}$$

$$s) \log_2 \sqrt[3]{\frac{1}{8}} = \log_2 \left(\frac{1}{2^3} \right)^{\frac{1}{3}} = \log_2 \frac{1}{2} = \log_2 2^{-1} = -1$$

$\left(\sqrt[3]{8} = 2 \right)$

$$t) \log_2 \sqrt[5]{\frac{1}{8}} = (2^{-3})^{\frac{1}{5}} = \log_2 2^{-\frac{3}{5}} = -\frac{3}{5}$$

$$u) \log_2 \sqrt[5]{\left(\frac{1}{8} \right)^7} = \log_2 \left((2^{-3})^7 \right)^{\frac{1}{5}} = \log_2 2^{-\frac{21}{5}} = -\frac{21}{5}$$

$= \frac{\log_2 2^{-21/5}}{-21/5}$

2

a) $\log_2 10 = ? \longrightarrow 2^x = 10 \quad / \log$

$$\log 2^x = \log 10 = 1$$

$$x \cdot \log 2 = 1$$

$$x = \frac{1}{\log 2} \approx 3.32$$

b) $\log_3 4718.823 = ? \quad 3^x = 4718.823 \quad / \log$

$$x = \frac{\log 4718.823}{\log 3} \approx 7.7$$

c) $7^x = 777 \Rightarrow x = \frac{\log 777}{\log 7}$

d) $x = \frac{\log 759375}{\log 15}$

3 a) $10^x = 20 \quad / \log$

$$x \cdot \log 10 = \log 20$$

$$x = \frac{\log 20}{\log 10}$$

b) $7^x = 30$

$$x = \frac{\log 30}{\log 7}$$

c) $3 \cdot 10^{-x+2} = 30 \quad / :3$
 $10^{-x+2} = 10 \quad / \log$

$$\underbrace{(-x+2)}_1 \cdot \underbrace{\log 10}_1 = \underbrace{\log 10}_1$$

$$-x + 2 = 1$$

$$x = 1$$

d) $2 \cdot 3^{2x-1} = 12 \quad / :2$
 $3^{2x-1} = 6 \quad / \log$

$$(2x-1) \log 3 = \log 6$$

$$2x-1 = \frac{\log 6}{\log 3}$$

$$x = \frac{1}{2} \left(\frac{\log 6}{\log 3} + 1 \right)$$

(4)

a) $\log x = 3.3 \quad | \cdot 10^x$

$$10^{\log x} = 10^{3.3}$$

$$\underline{\underline{x = 10^{3.3}}}$$

b) $4 \cdot \log x = 10 \quad | : 4$

$$\log x = \frac{5}{2}$$

$$10^{\log x} = 10^{5/2}$$

$$\underline{\underline{x = 10^{5/2}}}$$

c) $\frac{1}{3} \cdot \log x = 2 \quad | \cdot 3$

$$\log x = 6$$

$$10^{\log x} = 10^6$$

$$\underline{\underline{x = 10^6}}$$

d) $\log_2 x = 1.75$

$$2^{\log_2 x} = 2^{1.75}$$

$$\underline{\underline{x = 2^{1.75}}}$$

e) $\log(x-2) = 5$

$$10^{\log(x-2)} = 10^5$$

$$x-2 = 10^5$$

$$\underline{\underline{x = 10^5 + 2}}$$

f) $\log(x-3) = 4.4$

$$10^{\log(x-3)} = 10^{4.4}$$

$$x-3 = 10^{4.4}$$

$$\underline{\underline{x = 10^{4.4} + 3}}$$

5

$$-10 : 2^{10} = 1024 \approx 1000 \rightarrow 20 \text{ m}^2$$

⋮

$$-1 : 10'000$$

$$\text{heute} : 20'000 \text{ m}^2$$

$$+1 : 40'000$$

$$+2 : 80'000$$

$$+3 : 160'000 \text{ m}^2$$

6

$$1064.33 = 1000 (1 + 0.005)^x \quad / : 1000$$

$$1.06433 = (1.005)^x \quad / \log$$

$$\log(\quad) = x \cdot \log 1.005$$

$$x = \frac{\log 1.06433}{\log 1.005} \approx 12.5 \text{ J.}$$

7

$$5'802.70 = 5000 \cdot (1+p)^{10} \quad / : 5000$$

$$\frac{5802.70}{5000} = (1+p)^{10} \quad / \sqrt[10]{\quad}$$

$$p = \sqrt[10]{\frac{5802.70}{5000}} - 1 \approx 1.5\%$$

8

$$2 = 1 \cdot (1+p)^n$$

$$2 = 1.03^x \quad / \log$$

$$x = \frac{\log 2}{\log 1.03} \approx 23.45 \text{ J.}$$

$$(9) \quad 2 = 1(1+p)^{10} \quad | : 1 \quad \sqrt[10]{\quad}$$

$$\sqrt[10]{2} = 1+p$$

$$p = \sqrt[10]{2} - 1 \approx 7.18\%$$

$$(10) \quad 2'320.80 = 2000 \cdot (1.02)^x (1.01)^x$$
$$\frac{2320.80}{2000} = \frac{(1.02 \cdot 1.01)^x}{\log \left(\frac{2320.80}{2000} \right)}$$
$$x = \frac{\log \left(\frac{2320.80}{2000} \right)}{\log(1.02 \cdot 1.01)} \approx 5 \text{ Jahre}$$

$$(11) \quad 10^6 = 0.01 \cdot (1.01)^x \quad | : 0.01$$
$$10^8 = (1.01)^x$$
$$x = \frac{\log 10^8}{\log 1.01} = \frac{8}{\log 1.01} = \underline{\underline{1851.26 \text{ J}}}$$