Gest BM 7M, 18. 10. 13

(1) $pH = 3$; $pH = -\log[H^+]$

a) $pH = 3$: $-\log[H^+] = 3$

$[H^+] = 10^{-3}$

$pH = 5.3$ $-\log[H^+] = 5.3$

$[H^+] = 10^{-5.3}$

$$\frac{10^{-3}}{10^{-5.3}} \cong \underline{\underline{199.53 \cong 200\text{-fach}}}$$

b) $pH = 3$: $[H^+] = 10^{-3}$ mol/l

1'500-fach verdünnen:

$$[H^+] = \frac{10^{-3}}{1'500}$$

$$pH = -\log\left(\frac{10^{-3}}{1500}\right) \cong \underline{\underline{6.176 \cong 6.18}}$$

(2) $L = 10 \cdot \log(I/I_0)$

$$90\,dB = 10 \cdot \log\left(\frac{100\,W}{I_0}\right)$$

Hier kann I_0 berechnet werden:

$$9 = \log\left(\frac{100}{I_0}\right)$$

$$10^9 = \frac{100}{I_0} \quad / \cdot I_0 : 10^9$$

$$I_0 = \frac{100}{10^9} = 10^{-7}$$

$$L = 10 \cdot \log\left(\frac{20}{10^{-7}}\right) = \underline{\underline{83.01\,dB}}$$

20 Watt : $L = 10 \cdot \log\left(\frac{20}{10^{-7}}\right) \hat{=} \underline{\underline{83.01\,dB}}$

Alternativ, ohne I_0 zu berechnen:

1 Anlage $\hat{=}$ 100 Watt $\longrightarrow$ I

$\frac{1}{5}$ Anlage $\hat{=}$ 20 Watt $\longrightarrow$ $\frac{1}{5} I$

$$90\,dB = 10 \cdot \log\left(\frac{I}{I_0}\right)$$

$$10^{9} = \frac{I}{I_0}$$

$$\frac{10^{9}}{5} = \frac{\frac{1}{5} I}{I_0}$$

$$L = 10 \cdot \log\left(\frac{10^{9}}{5}\right) = \underline{\underline{83.01\,dB}}$$

b) Mit $I_0 = 10^{-7}$ Watt:

$$115\,dB = 10 \cdot \log\left(\frac{I}{10^{-7}}\right)$$

$$11.5 = \log(\text{"})$$

$$10^{11.5} = \frac{I}{10^{-7}} \quad \Big/ \cdot 10^{-7}$$

$$10^{11.5} \cdot 10^{-7} = I$$

$$10^{11.5-7} = 10^{4.5} = \underline{\underline{31'622.777\,\text{Watt}}}$$

b) ohne I_0 zu kennen:

$$90 = 10 \cdot \log\left(I/I_0\right)$$

$$10^9 = I/I_0 \rightarrow \text{1 Anlage à 100 Watt}$$

$$115 = 10 \cdot \log\left(\frac{x \cdot I}{I_0}\right) \rightarrow \text{x Anlagen à 100}$$

$$115 = 10 \cdot \log\left(x \cdot 10^9\right), \text{ da } \frac{I}{I_0} = 10^9$$

$$11.5 = \log\left(x \cdot 10^9\right)$$

$$10^{11.5} = x \cdot 10^9$$

$$\frac{10^{11.5}}{10^9} = x = 10^{2.5} \cong 316.22777$$

316.22... Anlagen à 100 Watt

$$\Rightarrow \underline{\underline{31'622.777 \text{ Watt}}}$$

a) ohne I_0 zu kennen:

$$100\,W \mathrel{\hat{=}} 90\,dB = 10 \cdot \log\left(\frac{I}{I_0}\right)$$

$$9 = \log\left(I/I_0\right)$$

$$10^9 = I/I_0$$

$$\frac{I}{I_0} = 10^9 \Rightarrow \frac{1}{5} \cdot \frac{I}{I_0} = \frac{10^9}{5}$$

$$L = 10 \cdot \log\left(\frac{10^9}{5}\right)$$

(20 W sind $\frac{1}{5}$ von 100 W)

$$= 83.01\,dB$$

(3) $12\,\%$ Zuwachs $\longrightarrow$ $q = 1.12$

$$10 = 1 \cdot 1.12^{t/1} \quad ; \; t \text{ in Monaten}$$

$$10 = 1.12^{t} \quad \Big/ \log$$

$$\log 10 = 1 = t \cdot \log(1.12)$$

$$t = \frac{1}{\log(1.12)} \cong \underline{\underline{20.32 \text{ Monate}}}$$

(4) Zeit t in Monaten; $q = 1.02$

$$40 = 30 \cdot 1.02^{t} \quad \Big/ : 30$$

$$\frac{4}{3} = 1.02^{t} \quad \Big/ \log$$

$$\log 4/3 = t \cdot \log 1.02$$

$$t = \frac{\log 4/3}{\log 1.02} \cong \underline{\underline{14.527 \text{ Monate}}}$$

(5)

$$1000 \cdot 1.02^{t} = 10000 \cdot 0.95^{t} \quad \Big/ : 1000$$

$$1.02^{t} = 10 \cdot 0.95^{t} \quad \Big/ : 0.95^{t}$$

$$\frac{1.02^{t}}{0.95^{t}} = 10$$

$$\left(\frac{1.02}{0.95}\right)^{t} = 10 \quad \Big/ \log$$

$$t \cdot \log\left(\frac{1.02}{0.95}\right) = 1$$

$$t = \frac{1}{\log \frac{1.02}{0.95}} = \underline{\underline{32.387 \text{ Jahre}}}$$

⑥ $q = 10$; $\tau = 3$ Tage

a)
$$\begin{tikzcd}
1 \arrow[r] \arrow[r, bend right, "3"'] & 10 \arrow[r] \arrow[r, bend right, "3"'] & 1000 \arrow[r] \arrow[r, bend right, "3"'] & 1000 \arrow[r] \arrow[r, bend right, "3"'] & 10'000
\end{tikzcd}$$

12 Tage

$$10000 = 1 \cdot 10^{t/3} \quad / \log$$
$$\log 10'000 = \frac{t}{3} \cdot \log 10$$
$$4 = \frac{t}{3} \cdot 1 \quad / \cdot 3$$
$$\underline{\underline{12 = t}}$$

b)
$$80'000 = 1\% \cdot 8 \text{ Mio}$$
$$80'000 = 1 \cdot 10^{t/3} \quad / \log$$
$$\log 80'000 = \frac{t}{3} \cdot \log 10 \quad ; \quad \log 10 = 1$$
$$= \frac{t}{3}$$
$$3 \cdot \log 80'000 = t = \underline{\underline{14.7093 \text{ Tage}}}$$

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a)

$N(t=0) = N_0 = ?$

$N(t=2000) = 944.123090$

$N(t=8000) = 794.537691$

Nehme 4000 n.Chr.

als neuen Nullpunkt; $N_0 = 944.123090$

$$794.537691 = 944.123090 \cdot \left(\tfrac{1}{2}\right)^{\frac{6000}{\tau}}$$

$\longrightarrow$ 4000 n.Chr. bis 10'000 n.Chr. $\hat{=}$ 6000 Jahre

$$\log\left(\frac{794.537691}{944.123...}\right) = \frac{6000}{\tau} \cdot \log \tfrac{1}{2}$$

$$\tau = \frac{6000 \cdot \log \frac{1}{2}}{\log\left(\frac{794...}{944.123...}\right)} \underline{\underline{= 24'110 \text{ Jahre}}}$$

b) $N(t=-2000) = 944.123090 \cdot \left(\tfrac{1}{2}\right)^{\frac{-2000}{24110.0088}}$

$= 999.99...$

$\hat{=}$ $\underline{\underline{1 \text{ Tonne}}}$

c)
$$\frac{1}{100} = 1 \cdot \left(\tfrac{1}{2}\right)^{\frac{t}{24110}}$$

$$\frac{1}{100} = \left(\tfrac{1}{2}\right)^{\frac{t}{24110}} \quad / \log$$

$$-2 = \frac{t}{24110} \cdot \log \tfrac{1}{2}$$

$$\frac{-2 \cdot 24110}{\log \frac{1}{2}} = t \cong 160'183.373$$

Im Jahr $\underline{\underline{162'183.373}}$

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$T(t=5') = 900\,°C$

$T(t=8') = 757.32\,°C$

Nehme $t = 5'$ als neuen Nullpunkt:

$\Delta t = 8' - 5' = 3'$

$$757.32 = 900 \cdot \left(\frac{1}{2}\right)^{\frac{3}{\tau}} \quad \Big|\, : 900$$

$$\log\left(\frac{757.32}{900}\right) = \frac{3}{\tau} \cdot \log\left(\frac{1}{2}\right)$$

$$\tau = \frac{3 \cdot \log(1/2)}{\log\left(\frac{757.32}{900}\right)} \cong 12.04713$$

T halbiert in 12.04713 Minuten

Starttemp.: bez. $t = 5'$ als Nullpunkt

ist Start bei $t = -5$ min !

$$T(t=-5) = 900 \cdot \left(\frac{1}{2}\right)^{\frac{-5}{12.04713}} \cong 1'200\,°C$$

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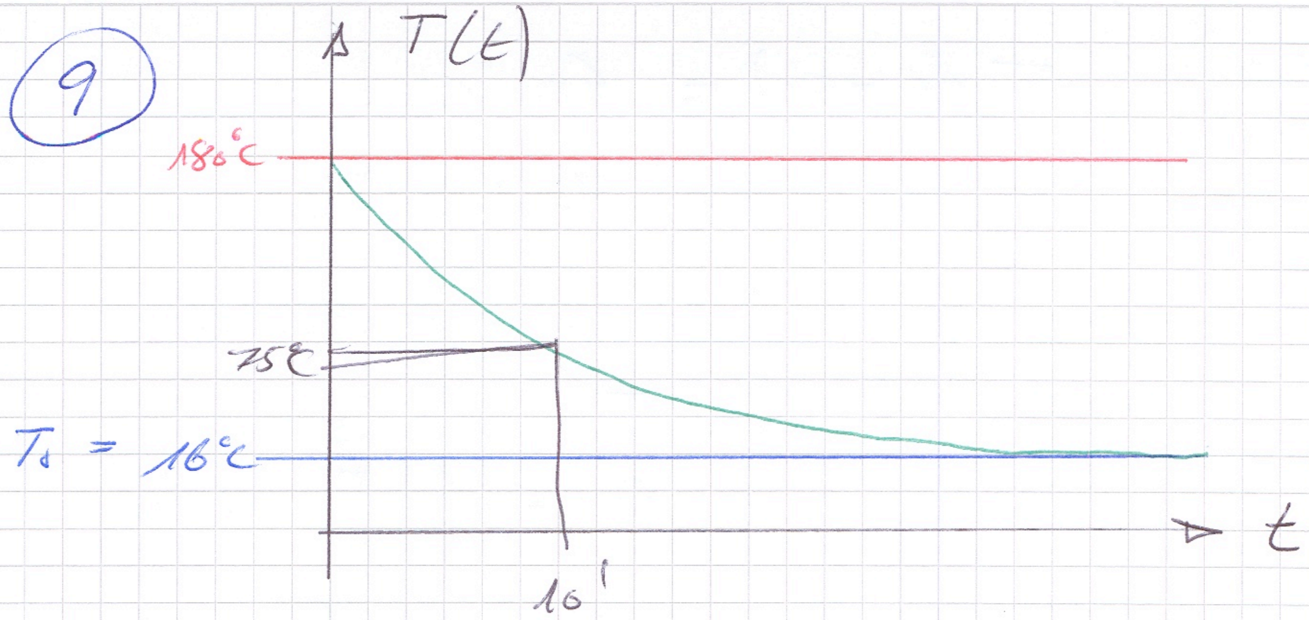

$$T(t) = \Delta T_0 \cdot \left(\tfrac{1}{2}\right)^{t/\tau} + T_0$$

$$\Delta T = 180°C - 16°C = 164°C$$

$$T(t) = 164 \cdot \left(\tfrac{1}{2}\right)^{t/\tau} + 16$$

$$T(t=10') = 75 = 164 \left(\tfrac{1}{2}\right)^{\frac{10}{\tau}} + 16 \quad \Big/ -16$$

$$59 = 164 \cdot \left(\tfrac{1}{2}\right)^{\frac{10}{\tau}} \quad \Big/ : 164$$

$$\frac{59}{164} = \left(\tfrac{1}{2}\right)^{10/\tau} \quad \Big/ \log$$

$$\log\left(\frac{59}{164}\right) = \frac{10}{\tau} \cdot \log(1/2)$$

$$\tau = \frac{10 \cdot \log(1/2)}{\log\left(\frac{59}{164}\right)} \cong 6.78$$

$$T(t) = 30 = 164 \cdot \left(\tfrac{1}{2}\right)^{t/\tau} + 16 \quad \Big/ -16$$

$$14 = \text{"} \qquad \text{"} \quad \Big/ : 164$$

$$\frac{14}{164} = \left(\tfrac{1}{2}\right)^{t/\tau} \quad \Big/ \log$$

$$\log\left(\frac{14}{164}\right) = \frac{t}{6.78\ldots} \cdot \log\frac{1}{2}, \quad t = \frac{6.78\ldots \cdot \log\left(\frac{14}{164}\right)}{\log 1/2}$$

$$\underline{24.071 \text{ Minuten}}$$