(1)

a) $T_{1/2} = 8.0207d \approx 8d$

10-mal Halbwertszeit $\longrightarrow$ 10 Halbierungen

$$\left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \approx \frac{1}{1000}$$

$\longrightarrow$ nach ca. 80 Tagen noch $1/1000$

b)

Anfang: N_0

nach 1 Tag: $N_0 \cdot q$

2 : $N_0 \cdot q \cdot q = N_0 \cdot q^2$

⋮

8 : $N_0 \cdot q^8 = \frac{1}{2} N_0$

$$q^8 = \frac{1}{2}$$

$$q = \sqrt[8]{\frac{1}{2}} = 2^{-\frac{1}{8}}$$

nach 1 Tag also: $\sqrt[8]{\frac{1}{2}} \cdot N_0$

2 : $\sqrt[8]{\frac{1}{2}} \cdot \sqrt[8]{\frac{1}{2}} \cdot N_0 = \sqrt[8]{\left(\frac{1}{2}\right)^2} N_0$

3 : $\sqrt[8]{\left(\frac{1}{2}\right)^3} N_0$

⋮

8 : $\sqrt[8]{\left(\frac{1}{2}\right)^8} N_0 = \frac{1}{2} N_0$

c) $\lambda = \frac{\ln n}{T_n} = \frac{\ln \frac{1}{2}}{8.0207} = - \frac{\ln 2}{8.0207d}$

$= \underline{\underline{-0.086'412/d}}$

↳ pro vorhandenem Atom und pro Tag zerfallen ~0.086 Atome

d) $N_0 = 1g \;;\; N(t) = 0.1g$

$N(t) = N_0 e^{\lambda t}$

$0.1g = 1g\, e^{\lambda t}$

$0.1 = e^{\lambda t} \quad / \ln$

$\ln 0.1 = \lambda t \quad ; \quad \ln 0.1 = -\ln 10$

$\ln \frac{1}{10} = \underbrace{\ln 1}_{0} - \ln 10$

$t = \frac{-\ln 10}{\lambda}$

$= \frac{-\ln 10}{-\frac{\ln 2}{8.0207d}} = \frac{8.0207 \cdot \ln 10}{\ln 2}$

$= \underline{\underline{26.644 \text{ Tage}}}$

e) $N(t = 365d) = N_0 \cdot e^{\lambda t} \quad / : N_0$

$\frac{N(t=365)}{N_0} = e^{\lambda t} = e^{365\lambda} = 2 \cdot 10^{-14}$

$= \underline{\underline{2 \cdot 10^{-12}\,\%}}$

↳ proz. Anteil

Bsp.: $N_0 = 100,\ N(t) = 15$

$\frac{N(t)}{N_0} = \frac{15}{100} = 0.15 = 15\%$

f)

$$A(t) = \lambda \cdot N(t)$$

$$A_0 = \lambda \cdot N_0 = \ ?$$

$$N_0 = \frac{1\,\mu g}{131} \cdot \underbrace{6.02 \cdot 10^{23}}_{N_A}$$

$$A = 3.97 \cdot 10^{14} \text{ pro Tag}$$

$$= 4.596 \cdot 10^{9} \text{ pro sec.}$$

$$= \underline{\underline{4.596 \cdot 10^{9}\ Bq}}$$

g)

$$A(t) = \lambda \cdot N(t) = 1000\,Bq \cdot 24 \cdot 3600$$

$$\lambda \cdot N_0 e^{\lambda t} = \quad \text{"}$$

$$N_0 = \frac{1\,\mu g}{131} \cdot N_A$$

$$e^{\lambda t} = \frac{1000 \cdot 24 \cdot 3600}{\lambda \cdot N_0} \quad \Big| \ Ln$$

$$\lambda t = Ln(\quad \text{"} \quad)$$

$$t = \frac{Ln\left(\frac{1000 \cdot 24 \cdot 3600}{\lambda \cdot N_0}\right)}{\lambda}$$

$$= 177.51 \text{ Tage}$$

h) $\sim 61\,MBq$ (Mega-Bq)

(2) Setze $N_0 = 765.4g \quad (t=0)$

$$N(t=1000s) = 410.16g$$

$$N(t=1000s) = 410.16 = 765.4 \cdot e^{\lambda t}$$

$$\frac{410.16}{765.4} = e^{\lambda t} \quad / Ln$$

$$Ln\left(\frac{410.16}{765.4}\right) = \lambda \cdot t \; ; \; t = 1000s$$

$$\frac{Ln\left(\frac{410.16}{765.4}\right)}{1000} = \lambda = -0.000'624/s$$

$$T_{1/2} = \frac{-Ln 2}{\lambda} = \underline{\underline{1'111.07766\,s}}$$

(3) $A(t=0) = -7.241'472 \cdot 10^{22} / Jahr$

$m = 1'000\,kg$

$$A = \lambda \cdot N_0 \; ; \; N_0 = \frac{10^6 g}{239} \cdot 6.02 \cdot 10^{23}$$

$$\lambda = \frac{A}{N_0} = -0.000'028'749$$

$$T_{1/2} = -\frac{Ln 2}{\lambda} \cong \underline{\underline{24'110\text{ Jahre}}}$$

(4) $T(t=0) = T_w = 92°C = T_{start}$

$T_k = 20°C = T_{end}$

$T(t=5') = 76°C$

$T(t) = \Delta T \cdot e^{\lambda t} + T_{end}$

$\Delta T = T_{start} - T_{end}$

$\Delta T = 72°C$

$T(t=5) = 72 \cdot e^{\lambda \cdot 5} + 20 = 76 \quad / -20$

$72 \cdot e^{5\lambda} = 56$

$e^{5\lambda} = \frac{56}{72} = \frac{7}{9} \quad / \ln$

$5\lambda = \ln(7/9)$

$\lambda = \frac{\ln(7/9)}{5}$

$\hat{=} \underline{\underline{-0.050'263 / m}}$

b) $T(t=10) = 72e^{10\lambda} + 20 \hat{=} \underline{\underline{63.556°C}}$

c) $T(t) = 72e^{\lambda t} + 20 = 21 \quad / -20$

$72e^{\lambda t} = 1 \quad / : 72$

$e^{\lambda t} = \frac{1}{72} \quad / \ln$

$\lambda \cdot t = -\ln(72)$

$t = -\frac{\ln(72)}{\lambda} = \underline{\underline{85.086'}}$

5

$$T(t) = \Delta T \cdot e^{\lambda t} + T_{end}$$

$$\Delta T = T_{start} - T_{end}$$

$$T(10') = 80\,°C$$

$$T(20') = 48\,°C$$

Wähle 10' als Nullpunkt, d.h.

$$T(0') = 80\,°C = T_{start}$$

$$T(10') = 48°\,C$$

$$T_{end} = 12\,°C$$

$$T_{start} - T_{end} = \Delta T = 68°$$

$$\hookrightarrow \quad T(t) = 68 e^{\lambda t} + 12$$

$$T(t=10) = 68 e^{\lambda \cdot 10} + 12 = 48° \quad / -12$$

$$68 e^{10\lambda} = 36 \quad /:68$$

$$e^{10\lambda} = \frac{36}{68} = \frac{9}{17} \quad / \ln$$

$$10\lambda = \ln(9/17)$$

$$\lambda = \frac{1}{10} \cdot \ln(9/17)$$

$$\cong -0.0636\ °C/min$$

$$T(t) = 68 e^{\lambda t} + 12 = 20° \quad / -12$$

$$68 e^{\lambda t} = 8$$

$$e^{\lambda t} = 8/68 = 2/17 \quad / \ln$$

$$\lambda t = \ln(2/17)$$

$$t = \frac{\ln(2/17)}{\lambda}$$

$$\cong 33.65' ; \underline{\underline{43.65'}}$$

T zu Beginn: (nicht in Aufgabe gefragt)

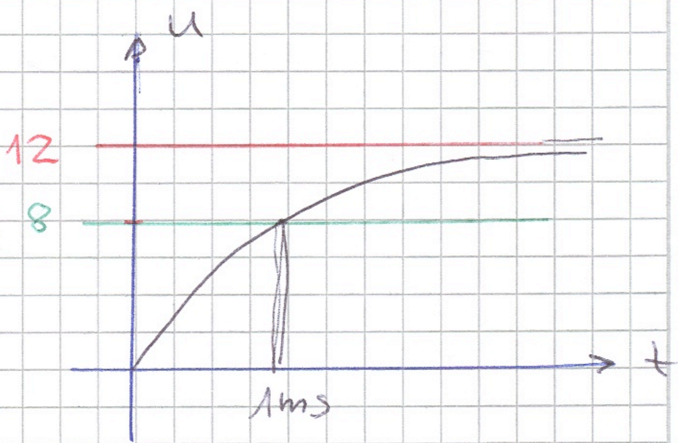
$$T(t = -10') = 68e^{\lambda \cdot (-10)} + 12$$

$$= \underline{\underline{140.44\,°C}}$$

6

$$u(t) = \Delta u e^{\lambda t} + u_{end}$$

$$\left.\begin{array}{l} u_{start} = 0 \\ u_{end} = 12V \end{array}\right\} \Delta u = -12V$$



$$\hookrightarrow \quad u(t) = -12e^{\lambda t} + 12$$

$$u(t = 1ms) = -12e^{\lambda \cdot 1} + 12 = 8$$

(Zeit in Millisekunden!)

$$-12e^{\lambda} = -4 \quad | :(-12)$$

$$e^{\lambda} = \frac{1}{3} \quad | \ln$$

$$\lambda = -\ln(3)$$

$$u(t) = -12e^{\lambda t} + 12 = 0.95 \cdot 12 = 11.4 \quad | -12$$

$$-12e^{\lambda t} = -0.6 \quad | :(-12)$$

$$e^{\lambda t} = \frac{-0.6}{-12} = 0.05 \quad | \ln$$

$$\lambda t = \ln(0.05)$$

$$t = \frac{\ln(0.05)}{\lambda} = \underline{\underline{2.73ms}}$$

b) never ever ....

7

$$\left.\begin{array}{l} U_{start} = 50V \\ U_{end} = 0V \end{array}\right\} \Delta U = 50V$$

$$u(t) = 50e^{\lambda t}$$

$$u(t = 0.25\mu s) = 50e^{\lambda \cdot 0.25} = 30 \quad / :50$$

$$e^{0.25 \cdot \lambda} = \frac{3}{5} \quad / \ln$$

$$0.25\lambda = \ln(3/5)$$

$$\lambda = \frac{\ln(3/5)}{0.25}$$

$$\cong -2.043'303 \ \text{Volt}/\mu s$$

10% $u = 5V$:

$$u(t) = 50e^{\lambda t} = 5$$

$$e^{\lambda t} = \frac{1}{10} \quad / \ln$$

$$\lambda t = -\ln(10)$$

$$t = -\frac{\ln(10)}{\lambda}$$

$$\cong \underline{\underline{1.127\mu s}}$$

1% $u = 0.5V$

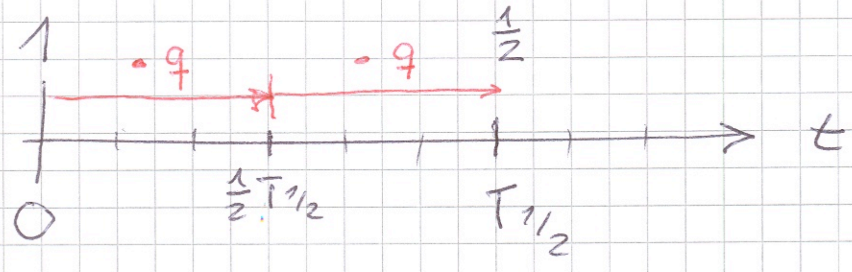
$$u(t) = 50e^{\lambda t} = 0.5$$

$$t = -\frac{\ln(100)}{\lambda}$$

$$= \underline{\underline{2.254\mu s}}$$

(8) Vorüberlegung:

innerhalb $T_{1/2}$ halbiert sich Menge.
Wie weit verringert sie sich innerhalb der halben Halbwertszeit, d.h. $\frac{1}{2} T_{1/2}$?



Innerhalb gleich langer Zeitabschnitte verringert sich die Menge mit dem ~~im~~ gleichen Faktor q

$$N\left(t = \tfrac{1}{2} T_{1/2}\right) = N_0 \, q$$

$$N\left(t = T_{1/2}\right) = \underbrace{N\left(t = \tfrac{1}{2} T_{1/2}\right)}_{N_0 q} \cdot q$$

$$\Longrightarrow \quad N\left(t = T_{1/2}\right) = N_0 \cdot q^2 = \tfrac{1}{2} N_0$$

$$q^2 = \frac{1}{2}$$

$$q = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Bis zur halben Halbwertszeit Abnahme um Faktor $\frac{\sqrt{2}}{2}$

Analog: $N\left(t = T_{1/2}\right) = N_0 \cdot \overbrace{q \cdot q \cdot \ldots \cdot q}^{10 \times}$

$$= N_0 \cdot q^{10} = \frac{1}{2}$$

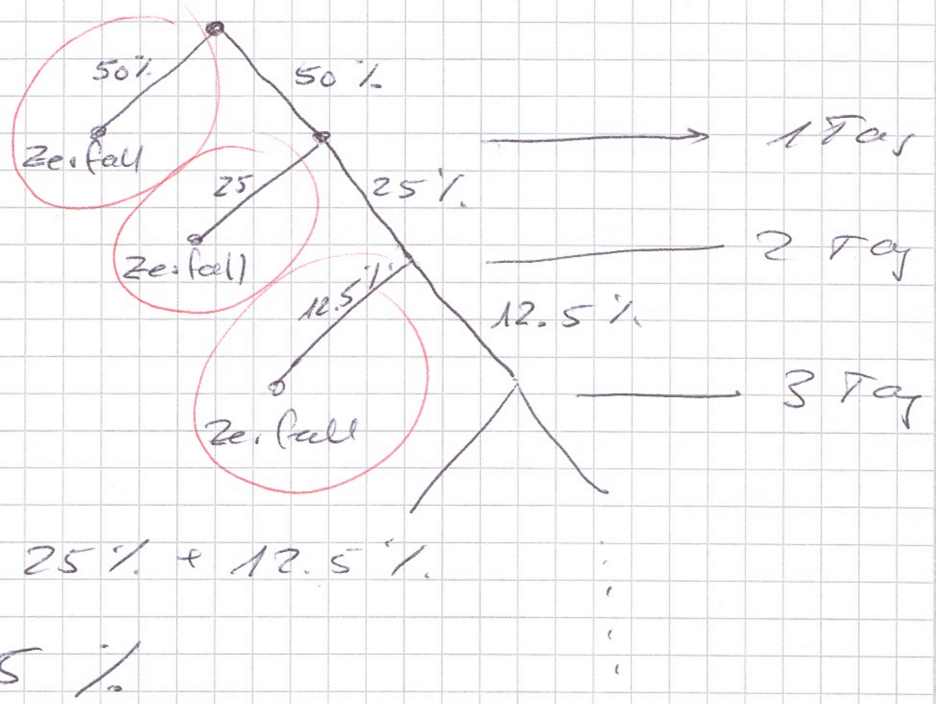
$$q = \frac{1}{\sqrt[10]{2}} = \underline{\underline{2^{-1/10}}}$$

b) Ja; in 2h Abnahme auf $\frac{1}{10}$

$\hookrightarrow \quad T_{1/10} = 2h$

Innerhalb 2h reduziert sich die Menge auf 1/10, egal wo das 2-stündige Zeitfenster liegt!

c) Innerhalb $T_{1/2}$ ist Wahrsch. 50%, dass Atom zerfällt



$p = 50\% + 25\% + 12.5\%$

$= 87.5\%$

(9) $T_{1/2} = 5'736\,a$; $\lambda = -\ln 2 / T_{1/2}$

Konz. normal $= 3 \cdot 10^{10}$ pro Gramm C

10 Gramm ; 44'689 Zerf. während 48h

$\Rightarrow \frac{365}{2} \cdot 44689$ Zerf. pro Jahr

$$A = \lambda \cdot N_0$$

$$-\frac{365}{2} \cdot 44'689 = -\frac{\ln 2}{T_{1/2}} \cdot N_0$$

$$+\frac{T_{1/2} \cdot \frac{365}{2} \cdot 44'689}{\ln 2} = N_0 \doteq 6.749 \cdot 10^{10}$$

$\hookrightarrow$ $6.749 \cdot 10^{10}$ pro 10 g

$6.749 \cdot 10^{9}$ pro 1 g

$$N(t) = N_0 e^{\lambda t}$$

$$6.749 \cdot 10^{9} = 3 \cdot 10^{10} e^{\lambda t}$$

$$\frac{6.749 \cdot 10^{9}}{3 \cdot 10^{10}} = e^{\lambda t} \quad / \ln$$

$$\ln\left(\frac{6.749}{30}\right) = \lambda t$$

$$\frac{\ln(\ ``\ )}{\lambda} = t = -6'709.612$$

Jahre

10 $M = 220g$

$A(t = 240s) = 6'969.74\ Bq$

$A(t = 300s) = 95.76\ Bq$

$A(t) = A_0 e^{\lambda t}$

Wähle $t = 240s$ als neuen Nullpunkt

$\hookrightarrow \quad A(t=0) = A_0 = 6'969.74\,Bq$

$A(t=60s) = 95.76\ Bq$

$A(t=60) = 95.76 = 6'969.74\ Bq\ e^{\lambda \cdot 60}$

$$\frac{95.76}{6'969.74} = e^{60\lambda} \quad / \ln$$

$$\frac{\ln(\ \text{"}\ )}{60} = \lambda \cong -0.071'458$$

$$T_{1/2} = -\frac{\ln 2}{\lambda} \cong \underline{\underline{9.7\,s}}$$

b) $A_0 = A(t = -240s)$

$= 6'969.74 \cdot e^{\lambda(-240)}$

$= 1.956 \cdot 10^{11}\ Bq$

$A_0 = \lambda \cdot N_0 \Rightarrow N_0 = \frac{A_0}{\lambda}$

$$\frac{N_0}{6.02 \cdot 10^{23}} \cdot 220 = \underline{\underline{1\,ng\ (\text{Nanogramm})}}$$

(11) $S-31; \quad \lambda = -0.2695\,s^{-1}$

$N_0 = 1g$

$N(t=7s) = ?$

$N(t=7s) = 1g \cdot e^{\lambda \cdot 7s} = \underline{\underline{0.152g}}$

(12) $\lambda \cdot T_n = \ln(n)$

$T_{1/2}: \quad \lambda = -\frac{\ln 2}{T_{1/2}}$

$$e^{\lambda \cdot T_{1/2}} = e^{-\frac{\ln 2}{T_{1/2}} \cdot T_{1/2}} = e^{-\ln 2}$$

$$= \left(e^{\ln 2}\right)^{-1} = 2^{-1} = \underline{\underline{\frac{1}{2}}}$$

$T_n: \quad \lambda = \frac{\ln(n)}{T_n}$

$$e^{\lambda \cdot T_n} = e^{\frac{\ln(n)}{T_n} \cdot T_n} = e^{\ln(n)} = \underline{\underline{n}}$$

(13) Ra: Radium; $T_{1/2} = 5.7a$

$$x\,\% = 100\,\% \cdot e^{-\frac{\ln 2}{5.7} \cdot 50} \hat{=} \underline{\underline{0.223\,\%}}$$

(14) $T_{1/2} = 12.836d; \quad \lambda = -\frac{\ln 2}{12.836}$

$A = 7'500\,Bq = 24 \cdot 3600 \cdot 7500 \,/\, \text{pro Tag}$

$A = \lambda \cdot N_0 \Rightarrow N_0 = \frac{A}{\lambda}$

$$\frac{24 \cdot 3600 \cdot 7500}{-\frac{\ln 2}{12.836}} = \underline{\underline{11'999'944'937.1}}$$

15 $1\,\mu g$ Pu-239; $T_{1/2} = 24'110\,a$

$$A = N_0 \cdot \lambda \; ; \; N_0 = \frac{1\,\mu g}{239} \cdot N_A$$

$$= \frac{10^{-6}}{239} \cdot 6.02 \cdot 10^{23}$$

$$\cong 2{,}519 \cdot 10^{15}$$

$$A = 72.41 \cdot 10^{9} / \text{Jahr}$$

$$\underline{\underline{A \cong 2'296\ Bq}}$$

(16) Sr-90; $T_{1/2} = 28.78\,a$

$\lambda = -\ln 2 / T_{1/2}$

10 Jahre $\longrightarrow$ 10^{21} Atome zerfallen

$$N(t=10a) = N_0 e^{\lambda \cdot 10}$$

eisetzen

$$N_0 - N(t) = 10^{21} = \underbrace{\Delta N}_{\text{Differenz}}$$

$\hookrightarrow t = 10a$

$$N_0 - N(t) = \Delta N \Rightarrow N(t) = N_0 - \Delta N$$

$$\hookrightarrow \quad N(t) = N_0 - \Delta N = N_0 e^{10\lambda}$$

$$N_0 - N_0 \cdot e^{10\lambda} = \Delta N$$

$$N_0 \left(1 - e^{10\lambda}\right) = \Delta N$$

$$N_0 = \frac{\Delta N}{1 - e^{10\lambda}} \cong 4.672 \cdot 10^{21}$$

$$\frac{N_0}{6.02 \cdot 10^{23}} \cdot 90 = \underline{\underline{698.5\,mg}}$$

$$N(t) = N_0 e^{\lambda t}$$

$$= \frac{\Delta N}{1 - e^{\lambda t}} \cdot e^{\lambda t}$$

$$= \frac{\Delta N \cdot e^{\lambda t}}{1 - e^{\lambda t}} = \frac{\frac{\Delta N \cdot e^{\lambda t}}{e^{\lambda t}}}{\frac{1 - e^{\lambda t}}{e^{\lambda t}}}$$

$$= \frac{\Delta N}{e^{-\lambda t} - 1} = 3.67 \cdot 10^{21} \text{ Atome}$$

$\underline{\underline{0.549\,g = 549\,mg}}$

17 $T_2 = 5h$, $\lambda = +\frac{\ln 2}{5}$

$2h \longrightarrow + 10^9$ Bakterien

$N_0 - N(t) = -10^9$ $\left(N(t) > N_0 \;!\right)$

$\hookrightarrow$ No 16

$$N_0 = \Delta N \Big/ (1 - e^{2\lambda}) \cong \underline{\underline{3.13 \cdot 10^9}}$$

$$N(t = 2h) = \frac{\Delta N}{e^{-\lambda t} - 1} = \underline{\underline{4.13 \cdot 10^9}}$$