TBM5, 16.12.2016

1) a) $2^{x+2} + 10 \cdot 5^{x+1} = 2^{x+4} - 5^{x+2}$

$$10 \cdot 5^{x+1} + 5^{x+2} = 2^{x+4} - 2^{x+2}$$

$$10 \cdot 5^{1} \cdot 5^{x} + 5^{2} \cdot 5^{x} = 2^{4} \cdot 2^{x} - 2^{2} \cdot 2^{x}$$

$$5^{x}(5 \cdot 10 + 25) = 2^{x}(16 - 4)$$ 2

$$\frac{5^{x}}{2^{x}} = \left(\frac{5}{2}\right)^{x} = \frac{16-4}{5 \cdot 10 + 25} = \frac{12}{75} = \frac{4}{25}$$ 3

$$\left(\frac{5}{2}\right)^{x} = \frac{4}{25} = \left(\frac{2}{5}\right)^{2} = \left(\frac{5}{2}\right)^{-2}$$

$$\underline{\underline{x = -2}}$$ 4

b) $10^{x} - 10^{x-1} - 10^{x-2} + 10^{x-3} = \frac{891}{10}$

$$10^{x} - 10^{-1}10^{x} - 10^{-2}10^{x} + 10^{-3}10^{x} = \frac{891}{10}$$

$$10^{x}\left(1 - \frac{1}{10} - \frac{1}{100} + \frac{1}{1000}\right) = \frac{891}{10}$$ 2

$$\frac{1000}{1000} - \frac{100}{1000} - \frac{10}{1000} + \frac{1}{1000}$$

$$\frac{1000 - 100 - 10 + 1}{1000}$$

$$10^{x} \cdot \frac{891}{1000} = \frac{891}{10}$$ 3

$$\frac{10^{x}}{1000} = \frac{1}{10} \quad / \cdot 1000$$

$$10^{x} = 100 = 10^{2}$$

$$\underline{\underline{x = 2}}$$ 4

c) $3^{2x+3} + 9^{x+2} = \frac{4}{27}$

$3^3 \cdot 3^{2x} + 9^{x+2} = \frac{4}{27}$ 1 $\left(3^{2x} = (3^2)^x = 9^x\right)$

$3^3 \cdot 9^x + 9^2 \cdot 9^x = \frac{4}{27}$ 2

$27 \cdot 9^x + 81 \cdot 9^x = \frac{4}{27}$ 3

$108 \cdot 9^x = \frac{4}{27}$

$9^x = \frac{4}{27 \cdot 108} = \frac{1}{27 \cdot 27} = \frac{1}{3^3 \cdot 3^3} =$

$9^x = \frac{1}{3^6} = 3^{-6} = (3^2)^{-3}$

$9^x = 9^{-3}$

$\underline{\underline{x = -3}}$

d) $2^{2x} = 9 \cdot 2^x - 8$ $(a^m)^n = (a^n)^m$

$(2^x)^2 - 9 \cdot 2^x + 8 = 0$ 1 $z = 2^x$

$z^2 - 9z + 8 = 0$ 2

$(z-1)(z-8) = 0$

$z - 1 = 0$ $z - 8 = 0$

$z = 1$ $z = 8$ 3

$z = 2^x = 1$ $z = 2^x = 8 = 2^3$

$x = 0$ $x = 3$

$\underline{\underline{\mathbb{L} = \{0, 3\}}}$ 4

2

a) $\log(5) - \log(3) = \log(x+2) - \log(x)$

$$\log\left(\frac{5}{3}\right) = \log\left(\frac{x+2}{x}\right) \quad / \, 10^{x} \qquad 1$$

$$\frac{5}{3} = \frac{x+2}{x} \quad / \cdot x \cdot 3 \qquad 2$$

$$5x = 3(x+2) \qquad 3$$

$$5x = 3x + 6$$

$$2x = 6$$

$$\underline{\underline{x = 3}} \quad 3 \qquad \mathbb{D} = \{x \in \mathbb{R} \,/\, x > 0\} \quad 1$$

b) $\log(x) + \log(x-2) + \log(x+3) = \log(14x)$

$$\log[x(x-2)(x+3)] = \log(14x) \quad / \, 10^{x} \qquad 1$$

$$x(x-2)(x+3) = 14x \quad / : x \quad (x \neq 0) \qquad 2$$

$$(0 \notin \mathbb{D})$$

$$(x-2)(x+3) = 14$$

$$x^2 + x - 6 = 14 \quad / -14$$

$$x^2 + x - 20 = 0$$

$$(x+5)(x-4) = 0 \qquad 3$$

$$x_1 = -5 \qquad x_2 = 4$$

$$\mathbb{D} = \{x \in \mathbb{R} \,/\, x > 2\}$$

$$\underline{\mathbb{L} = \{4\}}$$

c) $\log(x-1) - \underbrace{1}_{1 = \log(10^1)} = \log(2) - \log(x-2)$

$\log(x-1) - \log(10) = \log(2) - \log(x-2)$

$\log\left(\frac{x-1}{10}\right) = \log\left(\frac{2}{x-2}\right) \quad / \; 10^{+}$

$\frac{x-1}{10} = \frac{2}{x-2} \quad / \cdot 10 \cdot (x-2)$

$(x-1)(x-2) = 20$

$x^2 - 3x + 2 = 20 \quad / -20$

$x^2 - 3x - 18 = 0$

$(x+3)(x-6) = 0$

$x_1 = -3 \qquad x_2 = 6$

$\mathbb{D} = \{x \in \mathbb{R} \, / \, x > 2\}$

$\mathbb{L} = \{6\}$

d) $\log(x) + 15 = 5 \cdot \log(7) + 3 \cdot \log_7(x)$

$$\log(x) - \underbrace{3 \cdot \log_7(x)} = 5 \cdot \log(7) - 15$$

$$\log_7(x) = \frac{\log(x)}{\log(7)}$$

$$\log(x) - 3\,\frac{\log(x)}{\log(7)} = 5\left(\log(7) - 3\right)$$

$$\log(x)\left[1 - \frac{3}{\log(7)}\right] = 5\left(\log(7) - 3\right)$$

$$\log(x) \cdot \frac{\cancel{(\log(7) - 3)}}{\log(7)} = 5\left(\cancel{\log(7) - 3}\right)$$

$$\frac{\log(x)}{\log(7)} = 5$$

$$\log(x) = 5 \cdot \log(7) = \log(7^5)$$

$$\underline{\underline{x = 7^5}} \qquad \mathbb{D} = \{x > 0\}$$

1P. für Basiswechsel