TBM 5A, 14.12.12

① a) $\log 100'000'000 = \log 10^8 = \underline{8}$

b) $\log_3 243 = \log_3 3^5 = \underline{5}$

c) $\log 0.\overset{000}{000}'000'001 = \log 10^{-12} = \underline{-12}$

d) $\log_2 (1024^{13}) = \log_2 (2^{10})^{13}$

$= \log_2 2^{130} = \underline{130}$

② a) $10^{\frac{1}{2}\cdot \log 100} = 10^{\frac{1}{2}\cdot 2} = 10^1 = \underline{10}$

$= (10^{\log 100})^{1/2} = \sqrt{100} = \underline{10}$

b) $10^{3\cdot \log x} = (10^{\log x})^3 = \underline{x^3}$

c) $e^{\ln 1/x} = \underline{1/x}$

d) $\ln(e^{-1}) = -1 \cdot \ln e = \underline{-1}$

(3) a) $10^{\frac{\log x}{5}} = 10^{\frac{1}{5}\cdot \log x} = \left(10^{\log x}\right)^{1/5}$

$= x^{1/5} = \sqrt[5]{x}$

b) $e^{-\ln x} = e^{-1\cdot \ln x} = \left(e^{\ln x}\right)^{-1}$

$= x^{-1} = 1/x$

c) $10^{-\frac{1}{3}\cdot \ln x} = \left(10^{\ln x}\right)^{-1/3} = \frac{1}{\sqrt[3]{10^{\ln x}}}$

kann nicht vereinfacht werden!

$10^x \longleftrightarrow \log x$

$e^x \longleftrightarrow \ln x$

d) $e^{3\cdot \ln x^5} = \left(e^{\ln x^5}\right)^3 = (x^5)^3 = x^{15}$

(4) $e^{\sqrt{\ln x}}$ bedeutet: $e^{(\ln x)^{1/2}}$

zwar ist: $\left(e^{\ln x}\right)^{1/2} = x^{1/2} = \sqrt{x}$

ABER: $e^{(\ln x)^{1/2}} \neq \left(e^{\ln x}\right)^{1/2}$!!

Allgemein: $(a^b)^c \neq a^{(b^c)}$

$(3^3)^3 \neq 3^{(3^3)}$

$27^3 \neq 3^{27}$!

(5) a) Fehler; sollte

$10^x + 10^{x+3} = 100.1$ heissen!

$10^x + 10^3 \cdot 10^x = 100.1$

$10^x (1 + 10^3) = 100.1$

$10^x \cdot 1001 = 100.1$

$10^x = \frac{100.1}{1001} = \frac{1}{10}$

$10^x = 1/10 \quad / \log$

$x = \log \frac{1}{10} = \underline{\underline{-1}}$

Test: $10^{-1} + 10^{-1+3}$

$= \frac{1}{10} + 10^2 = \underline{\underline{100.1}}$

b) $3^x = 2^{x+1} = 2 \cdot 2^x \quad / : 2^x$

$\frac{3^x}{2^x} = 2$

$\left(\frac{3}{2}\right)^x = 2 \quad / \ln$

$x \cdot \ln 3/2 = \ln 2$

$x = \frac{\ln 2}{\ln 3/2}$

(5) $10^x(1+10^4)$

$$10^x = \frac{100 \cdot 1}{10001}$$

$$x = \log\left(\frac{100 \cdot 1}{10001}\right)$$

(6) $\ln(x-3e) = 1 \quad / e^{x}$

$e^{\ln(x-3e)} = e^{1} = e$

$x - 3e = e \quad / + 3e$

$\underline{\underline{x = 4e}}$

einfacher: $\ln(?) = 1$

$\hookrightarrow \; ? = e$

denn $\ln(e) = 1$

$\Rightarrow \quad x - 3e = e \Rightarrow \underline{\underline{x = 4e}}$

b) $\log(x) = 1 - \log(x-3)$

$\log(x) + \log(x-3) = 1 \quad / 10^{x}$

$10^{\log(x) + \log(x-3)} = 10^{1} = 10$

$10^{\log x} \cdot 10^{\log(x-3)} = 10$

$x(x-3) = 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x_1 = 5$

$\underline{x_2 = -2}$

} nur $x_1 = 5$ ist Lösung, da $\log(x)$ für $x < 0$ nicht existiert!

(7) $\ln\left(\frac{x}{yz^2}\right)^s = s \cdot \ln\left(\frac{x}{yz^2}\right)$

$= s\left(\ln x - \ln(yz^2)\right)$

$= s\left(\ln x - (\ln y + \ln z^2)\right)$

$= s \cdot \ln x - s \cdot \ln y - 2s \cdot \ln z$