

TBM SA, 14.12.12

① a) $\log 100'000'000 = \log 10^8 = \underline{\underline{8}}$

b) $\log_3 243 = \log_3 3^5 = \underline{\underline{5}}$

c) $\log 0.\overset{000}{000}000'001 = \log 10^{-92} = \underline{\underline{-92}}$

d) $\log_2 (1024^{13}) = \log_2 (2^{10})^{13}$
 $= \log_2 2^{130} = \underline{\underline{130}}$

② a) $10^{\frac{1}{2} \cdot \log 100} = 10^{\frac{1}{2} \cdot 2} = 10^1 = \underline{\underline{10}}$
 $= (10^{\log 100})^{\frac{1}{2}} = \sqrt{100} = \underline{\underline{10}}$

b) $10^{3 \cdot \log x} = (10^{\log x})^3 = \underline{\underline{x^3}}$

c) $e^{\ln \frac{1}{x}} = \underline{\underline{\frac{1}{x}}}$

d) $\ln(e^{-1}) = -1 \cdot \ln e = \underline{\underline{-1}}$

$$\textcircled{3} \quad a) \quad 10^{\frac{\log x}{5}} = 10^{\frac{1}{5} \cdot \log x} = \left(10^{\log x}\right)^{1/5}$$

$$= x^{1/5} = \underline{\underline{\sqrt[5]{x}}}$$

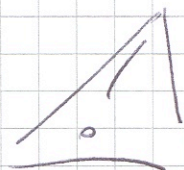
$$b) \quad e^{-\ln x} = e^{-1 \cdot \ln x} = \left(e^{\ln x}\right)^{-1}$$

$$= x^{-1} = \underline{\underline{\frac{1}{x}}}$$

$$c) \quad 10^{-\frac{1}{3} \cdot \ln x} = \left(10^{\ln x}\right)^{-1/3} = \frac{1}{\sqrt[3]{10^{\ln x}}}$$

↳ kann nicht vereinfacht werden!

$$10^x \leftrightarrow \log x$$

$$e^x \leftrightarrow \ln x$$


$$d) \quad e^{3 \cdot \ln x^5} = \left(e^{\ln x^5}\right)^3 = \left(x^5\right)^3 = \underline{\underline{x^{15}}}$$

$$\textcircled{4} \quad e^{\sqrt{\ln x}} \text{ bedeutet: } e^{(\ln x)^{1/2}}$$

zwar ist: $\left(e^{\ln x}\right)^{1/2} = x^{1/2} = \sqrt{x}$

ABER: $e^{(\ln x)^{1/2}} \neq \left(e^{\ln x}\right)^{1/2} !!$

Allgemein: $(a^b)^c \neq a^{(b^c)}$

$$(3^3)^3 \neq 3^{(3^3)}$$

$$27^3 \neq 3^{27} !$$

5) a) Fehler; sollte

$$10^x + 10^{x+3} = 100 \cdot 1 \quad \text{heissen!}$$

$$10^x + 10^3 \cdot 10^x = 100 \cdot 1$$

$$10^x (1 + 10^3) = 100 \cdot 1$$

$$10^x \cdot 1001 = 100 \cdot 1$$

$$10^x = \frac{100 \cdot 1}{1001} = \frac{1}{10}$$

$$10^x = \frac{1}{10} \quad / \text{Log}$$

$$x = \text{Log} \frac{1}{10} = \underline{\underline{-1}}$$

$$\text{Test: } 10^{-1} + 10^{-1+3}$$

$$= \frac{1}{10} + 10^2 = \underline{\underline{100 \cdot 1}}$$

$$b) \quad 3^x = 2^{x+1} = 2 \cdot 2^x \quad / : 2^x$$

$$\frac{3^x}{2^x} = 2$$

$$\left(\frac{3}{2}\right)^x = 2 \quad / \text{Ln}$$

$$x \cdot \text{Ln} \frac{3}{2} = \text{Ln} 2$$

$$x = \underline{\underline{\frac{\text{Ln} 2}{\text{Ln} \frac{3}{2}}}}$$

5

$$10^x (1 + 10^4)$$

$$10^x = \frac{100.1}{10001}$$

$$x = \log\left(\frac{100.1}{10001}\right)$$

$$\textcircled{6} \quad \ln(x-3e) = 1 \quad / e^x$$

$$e^{\ln(x-3e)} = e^1 = e$$

$$x-3e = e \quad / +3e$$

$$\underline{\underline{x = 4e}}$$

einfacher: $\ln(?) = 1$

$$\hookrightarrow ? = e$$

denn $\ln(e) = 1$

$$\Rightarrow x-3e = e \Rightarrow \underline{\underline{x = 4e}}$$

$$b) \quad \log(x) = 1 - \log(x-3)$$

$$\log(x) + \log(x-3) = 1 \quad / 10^x$$

$$10^{\log(x) + \log(x-3)} = 10^1 = 10$$

$$10^{\log x} \cdot 10^{\log(x-3)} = 10$$

$$x(x-3) = 10$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x_1 = 5$$

$$\underline{\underline{x_2 = -2}}$$

} nur $x_1 = 5$ ist Lösung
da $\log(x)$ für
 $x < 0$ nicht existiert!

$$\begin{aligned} \textcircled{7} \quad \ln \left(\frac{x}{y^2 z^2} \right)^s &= s \cdot \ln \left(\frac{x}{y^2 z^2} \right) \\ &= s \left(\ln x - \ln(y^2 z^2) \right) \\ &= s \left(\ln x - (\ln y + \ln z^2) \right) \\ &= \underline{\underline{s \cdot \ln x - s \cdot \ln y - 2s \cdot \ln z}} \end{aligned}$$