TBM 5B, 25.11.2016

(1) a) $\ln\left(\frac{\sqrt[3]{a^2}}{\sqrt[5]{b^{10}c^{15}}}\right) = \ln\left(\frac{a^{2/3}}{b^2c^3}\right)$

$= \ln(a^{2/3}) - \ln(b^2c^3)$

$= \frac{2}{3}\ln(a) - \left(\ln(b^2) + \ln(c^3)\right)$

$= \frac{2}{3}\cdot\ln(a) - 2\ln(b) - 3\ln(c)$

b) $\log_2\left(\frac{1}{x\sqrt{x-1}}\right) = \underbrace{\log_2(1)}_{=0} - \log_2\left(x\cdot\sqrt{x-1}\right)$

$= -\left(\log_2\left(x\sqrt{x-1}\right)\right)$

$= -\left(\log_2(x) + \log_2\left((x-1)^{1/2}\right)\right)$

$= -\log_2(x) - \frac{1}{2}\log_2(x-1)$

(2) $2 - 3 \cdot \log(x) - 5\log(y)$

$= \log(10^2) - \log(x^3) - \log(y^5)$

$= \log(10^2) - (\log(x^3) + \log(y^5))$

$= \log(10^2) - \log(x^3 y^5)$

$= \log\left(\frac{10^2}{x^3 y^5}\right)$

(3) a) $\log_{1/4}(8^3) = -\frac{9}{2}$

$8^3 = \left(\frac{1}{4}\right)^x$

$(2^3)^3 = 2^9 = (2^2)^{9/2} = 4^{9/2} = \left(\frac{1}{4}\right)^{-9/2}$

$\hookrightarrow \log_{1/4}\left(\left(\frac{1}{4}\right)^{-9/2}\right) = -\frac{9}{2}$

b) $\log_{\sqrt[3]{a}}(a^{-1}) = -3$

~~$a^{1/3} = (a^{-1})^{-1/3} = (a^{-1/3})^{-1}$~~

$a^{-1} = (a^{1/3})^x \Rightarrow \frac{1}{3} \cdot x = -1$

$x = -3$

(4)

a) $6 \cdot 4^x - 5^{x+1} = 4^{x-1} - 2 \cdot 5^{x-1}$

$6 \cdot 4^x - 4^{-1} 4^x = 5 \cdot 5^x - 2 \cdot 5^{-1} \cdot 5^x$

$4^x \left(6 - \frac{1}{4}\right) = 5^x \left(5 - \frac{2}{5}\right)$

$4^x \cdot \frac{23}{4} = 5^x \cdot \frac{23}{5}$

$\frac{4^x}{5^x} = \left(\frac{4}{5}\right)^x = \frac{\frac{23}{5}}{\frac{23}{4}} = \frac{4}{5}$

$\underline{\underline{x = 1}}$

b) $2^x + 2^{x+1} + 2^{x+2} + 2^{x+3} = \frac{15}{2}$

$2^x (1 + 2 + 4 + 8) = \frac{15}{2}$

$2^x \cdot 15 = \frac{15}{2}$

$2^x = \frac{\frac{15}{2}}{\frac{15}{1}} = \frac{15}{30} = \frac{1}{2} = 2^{-1}$

$\underline{\underline{x = -1}}$

c) $5 \cdot 4^{x+3} + 6 \cdot 2^{2x+5} = 8$

$5 \cdot 4^3 4^x + 6 \cdot 2^5 \cdot 2^{2x} = 8$ $\qquad 2^{2x} = (2^2)^x = 4^x$

$320 \cdot 4^x + 6 \cdot 32 \cdot 4^x = 8$

$40 \cdot 4^x + 6 \cdot 4 \cdot 4^x = 1$

$4^x (40 + 24) = 1$

$4^x = \frac{1}{64} = 4^{-3}$

$\underline{\underline{x = -3}}$

(5) a) $1 + \log_3(x-7) = \log_3(x-1)$ $\quad D = \{x > 7\}$

$\log_3(3) + \log_3(x-7) = \log_3(x-1)$

$\log_3(3(x-7)) = \log_3(x-1) \quad / 3^x$

$3(x-7) = x-1$

$3x - 21 = x - 1 \quad / -x + 21$

$2x = 20$

$x = 10$

b) $\log(\log_3(\log_2(\log(x)))) = 0 \quad / 10^x$

$\log_3(\log_2(\log(x))) = 10^0 = 1 \quad / 3^x$

$\log_2(\log(x)) = 3^1 = 3 \quad / 2^x$

$\log(x) = 2^3 = 8$

$x = 10^8$

$D = \{x > 100\}$

c) $\log(x-2) = \log(27) - \log(x+4)$ $\quad D = \{x > 2\}$

$\log(x-2) + \log(x+4) = \log(27)$

$\log((x-2)(x+4)) = \log(27) \quad / 10^x$

$(x-2)(x+4) = 27$

$x^2 + 2x - 8 = 27 \quad / -27$

$x^2 + 2x - 35 = 0$

$(x-5)(x+7) = 0$

$x_1 = 5$

$x_2 = -7 \notin D$ $\quad \mathbb{L} = \{5\}$

$$\log_3\left(\log_2\left(\left|\log(x)\right|\right)\right) > 0$$

$$\log_2\left(\left|\log(x)\right|\right) > 1$$

$$\left|\log x\right| > 2$$

$$x > 10^2 = 100$$

$$8^3 = \left(\frac{1}{4}\right)^x$$

$$\left(2^3\right)^3 = 2^9 = \left(2^{-2}\right)^{-9/2}$$