

TBM 6E, 15.1.13

① a) $\log 10^{(3^3)} = \log 10^{27} = 27$

b) $\log_2 0.125 = \log_2 \frac{1}{8} = \log_2 2^{-3} = -3$

c) -10

d) $\log_3 (243^{20}) = \log_3 (3^5)^{20} = \log_3 3^{100} = 100$

② $\log x : x > 0 (!!!)$

③ $\log_3 729 : 3^x = 729 \quad / \ln$

$$x \cdot \ln 3 = \ln 729$$

$$x = \frac{\ln 729}{\ln 3}$$

④ a) $10^{3 \cdot \log \frac{1}{1000}} = 10^{-9} \quad (\log \frac{1}{1000} = -3)$

b) $10^{x \cdot \log x} = (10^{\log x})^x = x^x$

c) $\frac{1}{x}$

d) $\ln (e^{-1})^5 = \ln e^{-5} = -5$

$$(e^{-1})^5 = e^{-5}$$

||

$$\left(\frac{1}{e}\right)^5 = \frac{1}{e^5} = e^{-5}$$

5

$$a) 10^{\frac{\log x}{5}} = 10^{\frac{1}{5} \cdot \log x} = \left(10^{\log x}\right)^{\frac{1}{5}} = x^{\frac{1}{5}} = \sqrt[5]{x}$$

$$b) \ln(1) = 0!$$

$$e^0 = 1$$

$$c) 10^{-\frac{1}{5} \cdot \ln x^{20}} = \left(10^{\ln x^{20}}\right)^{-\frac{1}{5}} = \left(x^{20}\right)^{-\frac{1}{5}} = x^{-4} = \frac{1}{x^4}$$

$10^{-4 \cdot \ln x}$

$$d) e^{3 \cdot \ln x^5} = \left(e^{\ln x^5}\right)^3 = \left(x^5\right)^3 = x^{15}$$

6

$$10 \text{ W: } \frac{1}{8}$$

$$11 \text{ W: } \frac{1}{4}$$

$$12 \text{ W: } \frac{1}{2}$$

$$13 \text{ W: } 1$$

$$10 \text{ W: } \left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \approx \frac{1}{1000}$$

$$\rightarrow \approx 20 \text{ m}^2$$

$$\begin{aligned}
 \textcircled{7} \quad \ln\left(\frac{x}{y}\right) &= \ln x - \ln y \\
 &= -(\ln y - \ln x) \\
 &= -\left(\overset{\ln x}{\ln y - \ln x}\right) \\
 &= \underline{\underline{-\ln\left(\frac{y}{x}\right)}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \text{ a) } 2^{x+5} + 2^{x+3} + 2^{x-4} &= 40.0625 \\
 2^x (2^5 + 2^3 + 2^{-4}) &= 40.0625 \\
 32 + 8 + 0.0625 &
 \end{aligned}$$

$$2^x = 1$$

$$x = 0$$

$$\text{b) } 2^{x+1} - 3^x + 3^{x+1}$$

$$2 \cdot 2^x = 3^x (1+3)$$

$$2 \cdot 2^x = 4 \cdot 3^x \quad /: 2$$

$$2^x = 2 \cdot 3^x \quad /: 3^x$$

$$\frac{2^x}{3^x} = \left(\frac{2}{3}\right)^x = 2$$

$$x \cdot \ln \frac{2}{3} = \ln 2$$

$$x = \frac{\ln 2}{\ln \frac{2}{3}}$$

9

$$a) \ln(x-7) = 1$$

$$\Rightarrow x-7 = e$$

$$\underline{\underline{x = e + 7}}$$

$$b) \log(x+5) \neq 1 + \log(x-4)$$

$$10^{\log(x+5)} = 10^{1 + \log(x-4)}$$
$$= 10^1 \cdot 10^{\log(x-4)}$$

$$x+5 = 10 \cdot (x-4)$$

$$x+5 = 10x - 40$$

$$45 = 9x$$

$$\underline{\underline{x = 5}}$$

$$10 \cdot 10^{\frac{1}{\log x}} = 10^{(\log x)^{-1}}$$
$$\neq (10^{\log x})^{-1}$$

↪ geht nicht!

(11)

$$6^{\sqrt{x}} = 3^x \quad / \ln$$

$$\sqrt{x} \cdot \ln 6 = x \cdot \ln 3 \quad / : \sqrt{x}$$

$$\ln 6 = \frac{x}{\sqrt{x}} \cdot \ln 3$$

$$x = \sqrt{x} \cdot \sqrt{x}$$

$$\Rightarrow \frac{x}{\sqrt{x}} = \sqrt{x}$$

$$\ln 6 = \sqrt{x} \cdot \ln 3$$

$$\frac{\ln 6}{\ln 3} = \sqrt{x}$$

$$x = \left(\frac{\ln 6}{\ln 3} \right)^2$$
