

TBM SE, 1.3.13

① a) $\log 10^{(2^5)} = 2^5 \cdot \log 10 = 2^5 \cdot 1 = \underline{\underline{32}}$
 $= \log 10^{32} = 32$

b) $\log_3 \frac{1}{27} = \log_3 \frac{1}{3^3} = \log_3 3^{-3} = \underline{\underline{-3}}$

c) $\log 10^{-5} = \underline{\underline{-5}}$

d) $\log_4 64^2 = \log_4 (4^3)^2 = \log_4 4^{21} = \underline{\underline{21}}$
 $= 7 \cdot \log_4 64 = 7 \cdot 3 = 21$

② a) $\log 10^{\log 5 - \log 3} = \log (10^{\log 5} \cdot 10^{-\log 3})$
 $= \log (5 \cdot 3^{-1}) = \log \left(\frac{5}{3}\right)$
 $= (\log 5 - \log 3) \cdot \underbrace{\log 10}_{=1}$
 $= \log 5 - \log 3$
 $(= \log \frac{5}{3})$

b) $\ln \left((e^5)^4 \right)^3 = 3 \cdot \ln (e^5)^4 = 12 \cdot \ln e^5$
 $= \underline{\underline{60}}$
 $= \ln e^{60} = 60$

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$$\begin{aligned} \text{a) } \log \sqrt[3]{100} &= \log 100^{1/3} = \frac{1}{3} \cdot \log 100 = \underline{\underline{\frac{2}{3}}} \\ &= \log (10^2)^{1/3} = \log 10^{2/3} = \frac{2}{3} \cdot \log 10 \\ &= \underline{\underline{\frac{2}{3}}} \end{aligned}$$

$$\begin{aligned} \text{b) } \log_5 \sqrt[3]{125} &= \log_5 125^{1/3} = \frac{1}{3} \log_5 125 = \frac{1}{3} \cdot 3 = \underline{\underline{\frac{1}{1}}} \\ &= \log_5 (5^3)^{1/3} = \log_5 5^{3/3} = \underline{\underline{\frac{3}{3}}} \end{aligned}$$

$$\begin{aligned} \text{c) } \log x + \log \left(\frac{1}{x}\right) &= \log x + \log x^{-1} \\ &= \log x + (-1 \cdot \log x) \\ &= \log x - \log x = \underline{\underline{0}} \end{aligned}$$

$$= \log \left(x \cdot \frac{1}{x}\right) = \log 1 = \underline{\underline{0}}$$

$$\text{d) } \ln \frac{\sqrt[5]{e}}{e} = \ln \sqrt[5]{e} - \underbrace{\ln e}_{=1}$$

$$= \ln e^{1/5} - 1 = \frac{1}{5} - 1 = -\frac{4}{5}$$

$$\ln \frac{\sqrt[5]{e}}{e} = \ln \frac{e^{1/5}}{e^1} = \ln e^{\frac{1}{5}-1} = \ln e^{-4/5} = \underline{\underline{-\frac{4}{5}}}$$

$$\textcircled{4} a) 10^{3 \cdot \log 0.01} = (10^{\log 0.01})^3 \\ = (0.01)^3 = \underline{\underline{10^{-6}}}$$

$$\log 0.01 = -2; \quad 3 \cdot \log 0.01 = -6$$

$$\hookrightarrow 10^{3 \cdot (-2)} = \underline{\underline{10^{-6}}}$$

$$b) 10^{-2 \log x} = (10^{\log x})^{-2} = x^{-2} = \underline{\underline{\frac{1}{x^2}}}$$

$$c) x^2 \cdot e^{\ln \frac{1}{x^2}} = x^2 \cdot \frac{1}{x^2} = \underline{\underline{1}}$$

$$d) \log_a \sqrt{\frac{1}{a}} = \log_a \sqrt{a^{-1}} = \log_a (a^{-1})^{\frac{1}{2}} \\ = \log_a a^{-\frac{1}{2}} = \underline{\underline{-\frac{1}{2}}}$$

$$\textcircled{5} a) e^{-\frac{\ln x}{7}} = e^{-\frac{1}{7} \cdot \ln x} = (e^{\ln x})^{-\frac{1}{7}} = x^{-\frac{1}{7}} \\ = \underline{\underline{\frac{1}{\sqrt[7]{x}}}}$$

$$b) e^{\bar{u} \cdot \ln 1} = (e^{\ln 1})^{\bar{u}} = 1^{\bar{u}} = \underline{\underline{1}} \\ = e^{\bar{u} \cdot 0} = \underline{\underline{1}}, \text{ da } \ln 1 = 0!$$

$$(6) \quad \log_2 a = ? \rightarrow \log_2 a = x$$

$$\hookrightarrow 2^x = a \quad | \ln$$

$$x \cdot \ln 2 = \ln a$$

$$x = \log_2 a = \underline{\underline{\frac{\ln a}{\ln 2}}}$$

$$(7) \quad 2^{x+1} = 3^x + 3^{x-1}$$

$$2 \cdot 2^x = 3^x + \cancel{3^x} \cdot 3^{-1}$$

$$2 \cdot 2^x = 3^x + \frac{1}{3} \cdot 3^x$$

$$= 3^x \left(1 + \frac{1}{3} \right) = 3^x \cdot \frac{4}{3} \quad | : 2$$

$$2^x = \frac{2}{3} 3^x \quad | : 3^x$$

$$\frac{2^x}{3^x} = \frac{2}{3}$$

$$\left(\frac{2}{3} \right)^x = \frac{2}{3} \Rightarrow \underline{\underline{x = 1}}$$

$$\hookrightarrow x \cdot \ln \frac{2}{3} = \ln \frac{2}{3} \quad | : \ln \frac{2}{3}$$

$$\underline{\underline{x = 1}}$$

$$(8) \quad \ln(x-5) = \ln x - \ln 5$$

$$\ln(x-5) = \ln \frac{x}{5} \quad | e^x$$

$$e^{\ln(\dots)} = e^{\ln \frac{x}{5}}$$

$$x-5 = \frac{x}{5}$$

$$\frac{4}{5}x = 5$$

$$x = \underline{\underline{\frac{25}{4}}}$$