TBM 5E, 21.10.2016

(1) a) $\log(1000^3) = \log((10^3)^3)$

$= \log(10^9) = \underline{\underline{9}}$

b) $\log(10^{-77}) = \underline{\underline{-77}}$

c) $\log(1) = \underline{\underline{0}}$

d) $\log(0) = \underline{\underline{\text{nicht definiert}}}$

(2) a) $\log_5(125) = \log_5(5^3) = \underline{\underline{3}}$

b) $\log_4(16) = \log_4(4^{-2}) = \underline{\underline{-2}}$

c) $\log_{1/2}(16) = \log_{1/2}\left(\frac{1}{2}\right)^{-4} = \underline{\underline{-4}}$

d) $\log_{\sqrt{3}}(9^2) = \log_{3^{1/2}}(((3^{\frac{1}{2}})^4)^2)$

$= \log_{3^{1/2}}((3^{1/2})^8) = \underline{\underline{8}}$

(3) a) $\log(\sqrt[3]{100}) = \log(100^{1/3})$

$= \log((10^2)^{1/3}) = \log(10^{2/3}) = \underline{\underline{\frac{2}{3}}}$

b) $\log_a\left(\frac{1}{\sqrt{a}}\right) = \log_a(a^{-\frac{1}{2}}) = \underline{\underline{-\frac{1}{2}}}$

c) $\ln\sqrt[3]{e^{-2}} = \ln(e^{-2/3}) = \underline{\underline{-\frac{2}{3}}}$

d) $\log_{\sqrt[3]{x}}(x^2) = \log_{x^{1/3}}(x^{1/3})^6) = \underline{\underline{6}}$

(4) $x = \log_2(7)$

$\Updownarrow$

$2^x = 7 \quad / \log$

$x \cdot \log(2) = \log(7)$

$x = \dfrac{\log(7)}{\log(2)}$

5 a) $10^{\log(\sqrt[3]{x^2})} = \sqrt[3]{x^2}$ $\left(= x^{2/3}\right)$

b) $10^{\log(x)+3} = 10^3 \cdot 10^{\log x}$

$= 1000 \cdot x = 1000x$

c) $10^{\log(x) - \frac{1}{2}\log(y)}$

$= 10^{\log(x) - \log(y^{1/2})}$

$= 10^{\log\left(\frac{x}{y^{1/2}}\right)} = \frac{x}{\sqrt{y}}$

d) $e^{-\frac{1}{2}\ln(x^3) + \frac{1}{3}\ln(x)}$

$-\frac{3}{2}\ln(x) + \frac{1}{3}\ln(x)$ $\quad -\frac{3}{2} + \frac{1}{3} = -\frac{9}{6} + \frac{2}{6}$

$= e$

$= e^{-\frac{7}{6}\cdot\ln(x)} = e^{\ln(x^{-7/6})}$

$= x^{-7/6}$

6. a) $\log\left(\frac{a^2 b^{-3}}{c d^{-2}}\right) = \log(a^2 b^{-3}) - \log(c d^{-2})$

$= \log(a^2) + \log(b^{-3}) - (\log(c) + \log(d^{-2}))$

$= 2 \cdot \log(a) - 3 \cdot \log(b) - \log(c) - \log(d^{-2})$

$= \underline{\underline{2 \cdot \log(a) - 3 \cdot \log(b) - \log(c) + 2\log(d)}}$

b) $\ln\left(\frac{1}{a^2 - 2ab + b^2}\right) = \ln\left((a-b)^{-2}\right)$

$= \underline{\underline{-2 \cdot \ln(a-b)}}$

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a) $\log(x^5) - \log(x^3) + 2\log(y) - 3\log(z)$

$= 5\log(x) - 3\log(x) + 2\log(y) - 3\log(z)$

$= 2\log(x) \ldots$

$= \log(x^2) + \log(y^2) + \log(z^{-3})$

$= \log(x^2 \cdot y^2 \cdot z^{-3}) = \log\left(\frac{x^2 y^2}{z^3}\right)$

b) $2\ln(x+2) - \ln(x+3) - \ln(x+2)$

$= \ln((x+2)^2) - \ln(x+3) - \ln(x+2)$

$= \ln\left(\frac{(x+2)^2}{(x+2)(x+3)}\right) = \ln\left(\frac{x+2}{x+3}\right)$

einfacher:

$2\ln(x+2) - \ln(x+2) - \ln(x+3)$

$= \ln(x+2) - \ln(x+3)$

$= \ln\left(\frac{x+2}{x+3}\right)$

(8) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

$= -\left(\log_a(y) - \log_a(x)\right)$

$= -\log_a\left(\frac{y}{x}\right)$