TBM 5E, 18.11.2016

① a) $\log\left(\sqrt[5]{\frac{a^{15}b^{5}}{c^{25}}}\right) = \log\left(\left(\frac{a^{15}b^{5}}{c^{25}}\right)^{1/5}\right)$

$= \log\left(\frac{a^3 b}{c^5}\right) = \log(a^3 b) - \log(c^5)$

$= \underline{\underline{3\log(a) + \log(b) - 5\log(c)}}$

b) $\ln\left(\left(\frac{x^3}{y^4}\right)^{-5}\right) = \ln\left(\frac{x^{-15}}{y^{-20}}\right)$

$= \ln\left(\frac{y^{20}}{x^{15}}\right) = \underline{\underline{20 \cdot \ln(y) - 15\ln(x)}}$

② $1 + 2\cdot\log(x) - 3\cdot\log(y)$

$= \log(10) + \log(x^2) - \log(y^3)$

$= \log(10x^2) - \log(y^3)$

$= \underline{\underline{\log\left(\frac{10x^2}{y^3}\right)}}$

(3) a) $\log_{1/2}\sqrt{64} = \log_{1/2}(8)$

$$8 = \left(\frac{1}{2}\right)^x$$

$$\left(\frac{1}{2}\right)^{-3} = \left(\frac{1}{2}\right)^x \qquad x = -3$$

$$\log_{1/2}(8) = \log_{1/2}\left(\left(\frac{1}{2}\right)^{-3}\right) = \underline{\underline{-3}}$$

b) $\log_{\sqrt{a}}\left(\frac{1}{a^4}\right) =$

$$\frac{1}{a^4} = a^{-4} = \left(a^{1/2}\right)^{-8}$$

$$= \log_{\sqrt{a}}\left(\left(\sqrt{a}\right)^{-8}\right) = \underline{\underline{-8}}$$

④ a) $6\cdot 3^{x+1} + 3^{x+2} + 3^{x+3} = 6$

$$6\cdot 3^1 3^x + 3^2 3^x + 3^3 3^x = 6$$

$$3^x(18 + 9 + 27) = 6$$

$$3^x \cdot 54 = 6$$

$$3^x = \frac{6}{54} = \frac{1}{9} = 3^{-2}$$

$$3^x = 3^{-2}$$

$$\underline{\underline{x = -2}}$$

b) $2^{x+1} - 3^{x-1} = 3^x - 5\cdot 2^{x-1}$

$$2\cdot 2^x + 5\cdot \tfrac{1}{2}\cdot 2^x = 3^x + 3^{-1}3^x$$

$$2^x\left(2 + \frac{5}{2}\right) = 3^x\left(1 + \frac{1}{3}\right)$$

$$2^x \cdot \frac{9}{4} = 3^x \cdot \frac{4}{3}$$

$$\frac{2^x}{3^x} = \frac{\frac{4}{3}}{\frac{9}{4}} = \frac{\cancel{16}\,8}{27}$$

$$\left(\frac{2}{3}\right)^x = \left(\frac{\cancel{16}\,8}{27}\right) = \left(\frac{8}{27}\right) = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$$

$$\underline{\underline{x = 3}}$$

c) $8^{x-1} - 2^{3x+1} + 15 = 0$

$$8^x\cdot 8^{-1} - (2^3)^x 2^1 + 15 = 0$$

$$8^x 8^{-1} - 8^x\cdot 2 + 15 = 0$$

$$8^x\left(\frac{1}{8} - 2\right) = -15$$

$$8^x = \frac{-15}{-\frac{15}{8}} = \frac{-\frac{15}{1}}{-\frac{15}{8}} = 8$$

$$8^x = 8$$

$$\underline{\underline{x = 1}}$$

(5) a) $\ln(2) = \ln(x+2) - \ln(x-3)$

$\mathbb{D} = \{x \in \mathbb{R} \mid x > 3\}$

$\ln(2) = \ln\left(\frac{x+2}{x-3}\right) \quad | \, e^x$

$2 = \frac{x+2}{x-3} \quad | \cdot x-3$

$2(x-3) = x+2$

$2x - 6 = x + 2$

$\underline{\underline{x = 8}}$

5b) $\log_3(\log(x)) = 1 \quad | \, 3^x$

$3^{\log_3(\log(x))} = 3^1$

$\log x = 3 \Rightarrow \underline{\underline{x = 10^3}}$

5c) $\log_2(x+1) = 4 - \log_2(x-5) \quad \mathbb{D} = \{x > 5\}$

$\log_2(x+1) = \log_2(2^4) - \log(x-5)$

$\log_2(x+1) = \log_2\left(\frac{2^4}{x-5}\right) \quad | \, 2^x$

$x+1 = \frac{16}{x-5} \quad | \cdot (x-5)$

$(x+1)(x-5) = 16$

$x^2 - 4x - 5 = 16 \quad | -16$

$x^2 - 4x - 21 = 0$

$(x+3)(x-7) = 0$

$x_1 = -3 \notin \mathbb{D}$

$x_2 = 7 \qquad \mathbb{L} = \{7\}$