TBM 6, 27.1.2017

(1) Pu-239; $N(t) = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{\tau}}$

$$N(t = 1000a) = 97.166\%$$

$$N(t = 1000\,a) = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{1000}{\tau}} = 97.166\%$$

$$N_0 = 100\%$$

$$100 \cdot \left(\frac{1}{2}\right)^{\frac{1000}{\tau}} = 97.166 \quad \Big| : 100$$

$$\left(\frac{1}{2}\right)^{\frac{1000}{\tau}} = 0.97166 \quad \Big| \ln$$

$$\frac{1000}{\tau} \cdot \ln(1/2) = \ln(0.97166)$$

$$\tau = \frac{1000 \cdot \ln(1/2)}{\ln(0.97166)}$$

$$\tau = 24'110.029'058\ldots$$

$$\tau \approx 24'110 \text{ Jahre}$$

(2) Problematik:

Ist $T_{1/2}$ klein, nimmt Menge schnell ab, strahlt dabei aber sehr stark.

Ist $T_{1/2}$ gross, ist Strahlung schwach, hält aber sehr lange an.

(3) C-14: natürlicher Gehalt $= 3 \cdot 10^{-8}\,\%$

1988 $= 2.752 \cdot 10^{-8}\,\%$

$T_{1/2} = 5'736\ a$

$N(t) = N_0 \cdot q^{t/\tau}$

$N(t) = 3 \cdot 10^{-8}\,\% \cdot \left(\frac{1}{2}\right)^{\frac{t}{5736}}$

$N(t) = 3 \cdot 10^{-8}\,\% \cdot \left(\frac{1}{2}\right)^{\frac{t}{5736}} = 2.752 \cdot 10^{-8}\,\%$

$3 \cdot \left(\frac{1}{2}\right)^{\frac{t}{5736}} = 2.752 \quad | :3$

$\left(\frac{1}{2}\right)^{\frac{t}{5736}} = \frac{2.752}{3} \quad | \ln$

$\frac{t}{5736} \cdot \ln(1/2) = \ln\left(\frac{2.752}{3}\right)$

$t = \frac{5736 \cdot \ln\left(\frac{2.752}{3}\right)}{\ln(1/2)}$

$t = 714.028'927' \ldots$

$t \cong 714\ a$

Anno 1275

(4) $2000: \quad 300'000 \text{ M.}; \quad 7\% \text{ p.a.}$

$5'700'000 \text{ C.}; \quad -0.5\% \text{ p.a.}$

a) $N(t) = 300'000 \cdot 1.07^{16} = \underline{\underline{885'649 \text{ M.}}}$

$N(t) = 5'700'000 \cdot 0.995^{16} = \underline{\underline{5'260'707 \text{ C.}}}$

b) $300'000 \cdot 1.07^t = 5'700'000 \cdot 0.995^t$

$$3 \cdot 1.07^t = 57 \cdot 0.995^t$$

$$\frac{1.07^t}{0.995^t} = \frac{57}{3} = 19$$

$$\left(\frac{1.07}{0.995}\right)^t = 19 \quad /\ln$$

$$t \cdot \ln('') = \ln(19)$$

$$t = \frac{\ln(19)}{\ln\left(\frac{1.07}{0.995}\right)}$$

$$t = 40.517'286\ldots$$

$$t = 40.52 \text{ Jahre}$$

$\underline{\underline{\text{Anno } 2041}}$

c)

$$300'000 \cdot (1.07)^t = 8'000'000$$

$$3 \cdot (1.07)^t = 80 \quad /:3$$

$$(1.07)^t = \frac{80}{3} \quad /\ln$$

$$t \cdot \ln(1.07) = \ln\left(\frac{80}{3}\right)$$

$$t = \frac{\ln(80/3)}{\ln(1.07)}$$

$$t = 48.529'115'\ldots$$

$$t \cong 48.53 \text{ Jahre}$$

Anno 2049

d)

$$\cancel{21'000 \cdot x = 8'000'000}$$

$$\cancel{x = 380.952'380}\ldots$$

$$\cancel{x \cong 381 \text{ Jahre}}$$

$$N(t) = 21'000 \cdot x + 300'000 = 8'000'000$$

$$21x + 300 = 8'000$$

$$21x = 7'700$$

$$x = \frac{7'700}{21}$$

$$x \cong 366.\overline{6} \text{ Jahre}$$

$$\cong 367 \text{ Jahre}$$

Anno 2367

5 $N(t) = N_0 \cdot 2^{t/\tau}$

$N(t=5d) = N_0 \cdot 2^{5/\tau} = 1'048'576$

$N(t=7d) = N_0 \cdot 2^{7/\tau} = 268'435'456$

$$\frac{\cancel{N_0} \cdot 2^{5/\tau}}{\cancel{N_0} \cdot 2^{7/\tau}} = \frac{1048576}{268435456}$$

$$2^{\frac{5}{\tau} - \frac{7}{\tau}} = 2^{-2/\tau} = \frac{1}{256} \quad / \lg_2$$

$$-\frac{2}{\tau} = -8$$

$$\frac{2}{\tau} = 8$$

$$\tau = \frac{2}{8} = \frac{1}{4} d$$

$N_0 = 1'048'576 \cdot 2^{\frac{-5}{1/4}}$

$= \quad " \quad \cdot 2^{-20} = 1$

a) $\underline{\underline{\text{1 Bakterium wurde eingeschleppt}}}$

b) $\tau = 1/4\, d$

Verdoppelung in $\frac{1}{4}$ Tag $\Rightarrow$ $\underline{\underline{\text{16-fach in 1 Tag}}}$

⑥

$$T(t) = T_0 \cdot \left(\frac{1}{2}\right)^{t/\tau}$$

$$T(t=5') = T_0 \cdot \left(\frac{1}{2}\right)^{5/\tau} = 25\,°C$$

$$T(t=20') = T_0 \cdot \left(\frac{1}{2}\right)^{\frac{20}{\tau}} = 5\,°C$$

$$\frac{\cancel{T_0} \cdot \left(\frac{1}{2}\right)^{5/\tau}}{\cancel{T_0}\left(\frac{1}{2}\right)^{20/\tau}} = \frac{25}{5} = 5$$

$$\left(\frac{1}{2}\right)^{\frac{5}{\tau} - \frac{20}{\tau}} = \left(\frac{1}{2}\right)^{-\frac{15}{\tau}} = 5 \quad / \ln$$

$$-\frac{15}{\tau} \cdot \ln(1/2) = \ln(5)$$

$$\tau = \frac{-15 \cdot \ln(1/2)}{\ln(5)}$$

$$\tau = 6.460'148 \ldots$$

a) $$\underline{\underline{T_{1/2} \cong 6.46 \text{ Minuten}}}$$

$$T_0 = 25\,°C \cdot \left(\frac{1}{2}\right)^{\frac{-5}{\tau}} = 42.749'398'6 \ldots$$

b) $$\underline{\underline{T_0 \cong 42.75\,°C}}$$

c) $$T(t) = T_0 \cdot \left(\frac{1}{2}\right)^{t/\tau} = 3\,°C$$

$$\left(\frac{1}{2}\right)^{t/\tau au} = \frac{3}{T_0} \quad / \ln$$

$$\frac{t}{\tau} \cdot \ln(1/2) = \ln(3/T_0)$$

$$t = \frac{\tau \cdot \ln(3/T_0)}{\ln(1/2)} = 24.760'907 \ldots$$

$$\underline{\underline{t \cong 24.761' \cong 24.76' \cong 24.8'}}$$