TBM 6B, 3.2.2017

(1) $T(t) = T_0 \cdot q^{t/\tau}$; $T_0 = 85°C$

a) $T(t) = 850 \cdot \left(\frac{1}{10}\right)^{t/\tau}$; nähle wegen a) $q = \frac{1}{10}$

$T(t = 12') = 850 \cdot \left(\frac{1}{10}\right)^{\frac{12}{\tau}} = 160$

$\left(\frac{1}{10}\right)^{\frac{12}{\tau}} = \frac{160}{850} = \frac{16}{85}$ /Ln

$\frac{12}{\tau} \cdot \ln\left(\frac{1}{10}\right) = \ln\left(\frac{16}{85}\right)$

$\tau = \frac{12 \cdot \ln\left(\frac{1}{10}\right)}{\ln\left(\frac{16}{85}\right)}$

$\tau = 16.544'9021...$ min

$\tau \hat{=} 16.54$ Minuten

b) $T(t) = 850 \cdot \left(\frac{1}{10}\right)^{t/\tau} = 40$ /:850

$\left(\frac{1}{10}\right)^{\frac{t}{\tau}} = \frac{40}{850} = \frac{4}{85}$ /Ln

$\frac{t}{\tau} \cdot \ln\left(\frac{1}{10}\right) = \ln\left(\frac{4}{85}\right)$

$t = \frac{\tau \cdot \ln\left(\frac{4}{85}\right)}{\ln\left(\frac{1}{10}\right)}$

$t = 21.961'6236...$

$t \hat{=} 21.96$ Minuten

c) τ ist die Zeit, in der sich eine Menge auf einen Zehntel reduziert, egal wo man beginnt (bei Wahl von $q = 1/10$). τ = siehe a)

(2) $q = 2$; $\tau = 15\,h$

$$N(t) = N_0 \cdot 2^{\frac{t}{15}}$$

$$N(t = 168\,h) = N_0 \cdot 2^{\frac{168}{15}} = 100 \quad / \cdot 2^{-\frac{168}{15}}$$

$$N_0 = 100 \cdot 2^{-\frac{168}{15}}$$

$$N_0 = 42.507'3517\ldots\ \mu g$$

$$N_0 \approx 42.507\ \mu g$$

$(0.042'507\ mg)$

(3) $Cs-136: T_{1/2} = 13.16\,d = \tau_1$

$I-131: T_{1/2} = 8.0207\,d = \tau_2$

a) $N(t) = N_0 \cdot \left(\frac{1}{2}\right)^{t/\tau_1}$

Abnahme um 20 %: $N_0 = 5$; $N(t) = 4$

$$4 = 5 \cdot \left(\frac{1}{2}\right)^{\frac{t}{\tau_1}} \quad / : 5$$

$$\frac{4}{5} = \left(\frac{1}{2}\right)^{t/\tau_1} \quad / \ln$$

$$\ln\left(\tfrac{4}{5}\right) = \frac{t}{\tau_1} \cdot \ln\left(\tfrac{1}{2}\right)$$

$$t = \frac{\tau_1 \cdot \ln\left(\tfrac{4}{5}\right)}{\ln\left(\tfrac{1}{2}\right)} = 4.236\,573\,73\ldots$$

$$\underline{\underline{t \approx 4.24 \text{ Tage}}}$$

b) $$200 \cdot \left(\frac{1}{2}\right)^{\frac{t}{\tau_1}} = 300 \cdot \left(\frac{1}{2}\right)^{t/\tau_2} \quad / : 200$$

$$\left(\frac{1}{2}\right)^{\frac{t}{\tau_1}} = \frac{3}{2}\left(\frac{1}{2}\right)^{\frac{t}{\tau_2}} \quad / : \left(\frac{1}{2}\right)^{\frac{t}{\tau_2}}$$

$$\frac{\left(\frac{1}{2}\right)^{\frac{t}{\tau_1}}}{\left(\frac{1}{2}\right)^{\frac{t}{\tau_2}}} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_1} - \frac{t}{\tau_2}} = \frac{3}{2}$$

$$\left(\frac{1}{2}\right)^{t \cdot \left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)} = \frac{3}{2}$$

$$\left(\left(\frac{1}{2}\right)^{\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)}\right)^t = \frac{3}{2} \quad / \ln$$

$$t \cdot \ln\left(\left(\frac{1}{2}\right)^{\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)}\right) = \ln\left(\tfrac{3}{2}\right)$$

$$t = \frac{\ln\left(\frac{3}{2}\right)}{\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right) \cdot \ln\left(\frac{1}{2}\right)}$$

$$t = 12.014127\ldots$$

$$\underline{\underline{t \approx 12.01 \text{ Tage}}}$$

(4) $p(h) = p_0 \cdot \left(\frac{1}{2}\right)^{\frac{h}{5553.38}}$

$p_0 = 101'325\ Pa$

a) $p(h) = 101325 \cdot \left(\frac{1}{2}\right)^{\frac{h}{5553.38}} = 0.9 \cdot 101325$

$\left(\frac{1}{2}\right)^{\frac{h}{5553.38}} = 0.9 \quad / \ln$

$\frac{h}{5553.38} \cdot \ln\left(\frac{1}{2}\right) = \ln(0.9)$

$h = \frac{5'553.38 \cdot \ln(0.9)}{\ln\left(\frac{1}{2}\right)}$

$h = 844.130'939\ldots$

$h \approx 844.131$ Meter

b) $p(h = 8848m) = 101325 \cdot \left(\frac{1}{2}\right)^{\frac{8848}{5553.38}}$

$= 33'581.2131$

$\approx 33'581.21\ Pa$

≈ 0.334 bar

(5) $u(t) = u_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{\tau}}$

$u(t = 200\,ms) = u_0 \left(\frac{1}{2}\right)^{\frac{200}{\tau}} = 6.586\,V$

$u(t = 450\,ms) = u_0 \left(\frac{1}{2}\right)^{\frac{450}{\tau}} = 3.111\,V$

Solve: $u_0 = 12.000\,49\ldots \approx \underline{12\,Volt}$

$\tau = 231.048\,45\ldots \approx \underline{231.048\,ms}$

a) $q = \frac{1}{2} \Rightarrow \tau = T_{1/2} = \underline{\underline{231.048\,ms}}$

b) $\underline{\underline{u_0 = 12\,Volt}}$

c) $u(t) = u_0 \cdot \left(\frac{1}{2}\right)^{t/\tau} = 1\,Volt$

Solve: $t = 828.313\,748\ldots$

$\underline{\underline{t \approx 828.314\,ms}}$

(6)

$N_0 = 10\,g$

$N(t=4h) = 4.493\,g$

$N(t=9h) = 2.02\,g$

1) $N(t) = 10g \cdot \left(\frac{1}{2}\right)^{\frac{t}{\tau}}$

$N(t=4h) = 4.493 = 10 \cdot \left(\frac{1}{2}\right)^{\frac{4}{\tau}}$

$\tau = 3.465'456\ldots$

$\tau \approx 3.465\,h$

$N(t=9h) = 1.653\,g$

2) $N(t=9h) = 2.02 = 10 \cdot \left(\frac{1}{2}\right)^{\frac{9}{\tau}}$

$\tau = 3.900'201\ldots$

$\tau \approx 3.9$

$N(t=4h) = 4.912$

unterschiedliche τ → kein exp-Prozess