TBM 6E, 5.2.2016

Logarithmen I

1. a) $\log(100^2) = \log((10^2)^2) = \log(10^4) = \underline{4}$

b) $\log(-\frac{1}{10})$: ex. nicht, für $\log(x)$ gilt: $\underline{x > 0}$

c) $\log(1) = \log(10^0) = \underline{0}$

d) $\log(0)$: ex. nicht, siehe b)

2. a) $\log_2(32) = \log_2(2^5) = \underline{5}$

b) $\log_2(\frac{1}{8}) = \log_2(2^{-3}) = \underline{-3}$

c) $\log_3(9^7) = \log_3((3^2)^7) = \log_3(3^{14}) = \underline{14}$

d) $\log_5(\left(\frac{1}{\sqrt{5}}\right)^5) = \log_5((5^{-3})^5) = \log_5(5^{-15}) = \underline{-15}$

3. a) $\log\sqrt{1000} = \log((10^3)^{1/2}) = \log(10^{3/2}) = \underline{\frac{3}{2}}$

b) $\ln\sqrt[3]{e^4} = \ln(e^{4/3}) = \underline{\frac{4}{3}}$

c) $\log_2\sqrt{\frac{1}{8}} = \log_2((2^{-3})^{1/2}) = \log_2(2^{-3/2}) = \underline{-\frac{3}{2}}$

d) $\log_a((a^x)^{1/3}) = \log_a(a^{x/3}) = \underline{\frac{x}{3}}$

4. a) $10^{\log(3x^2)} = \underline{\underline{3x^2}}$

b) $10^{1+\log x} = 10^1 \cdot 10^{\log x} = 10 \cdot x = \underline{\underline{10x}}$

oder

$10^{1+\log x} = 10^{\log(10)+\log(x)}$

$= 10^{\log(10x)} = 10x$

c) $10^{2\cdot\log(a)-\log(b)} = 10^{2\cdot\log(a)} \, 10^{-\log b}$

$= 10^{\log(a^2)} \cdot 10^{\log(b^{-1})} = a^2 b^{-1} = \underline{\underline{\frac{a^2}{b}}}$

ALT:

$= 10^{\log(a^2)-\log(b)} = 10^{\log\left(\frac{a^2}{b}\right)} = \underline{\underline{\frac{a^2}{b}}}$

d) $\log\left(10^{(3^2)} \cdot 10\right) = \log\left(10^9 \cdot 10\right)$

$= \log\left(10^{10}\right) = \underline{\underline{10}}$

oder:

$= \log\left(10^{(3^2)}\right) + \log 10$

$= 3^2 + 1 = \underline{\underline{10}}$

5. a) $e^{-2\ln(x^2)} = \left(e^{\ln(x^2)}\right)^{-2} = (x^2)^{-2} = x^{-4}$

$= \frac{1}{x^4}$

oder:

$-2 \cdot \ln(x^2) = \ln\left((x^2)^{-2}\right) = \ln(x^{-4})$

$e^{\ln(x^{-4})} = x^{-4} = \frac{1}{x^4}$

b) $\log_a \frac{1}{\sqrt[3]{a^2}} = \log_a\left(\frac{1}{a^{2/3}}\right) = \log_a\left(a^{-2/3}\right) = -\frac{2}{3}$

c) $\rightarrow 4c; \quad \frac{a^2}{b}$

d) $10^{-\frac{2}{3}\log(x^2) + \frac{1}{3}\log(x)} =$

$-\frac{2}{3}\log(x^2) + \frac{1}{3}\log(x) = \log\left((x^2)^{-2/3}\right) + \log\left(x^{1/3}\right)$

$= \log\left(x^{-4/3} \cdot x^{1/3}\right) = \log(x^{-1})$

$10^{\log(x^{-1})} = x^{-1} = \frac{1}{x}$

oder:

$\left(10^{\log(x^2)}\right)^{-2/3} \cdot \left(10^{\log(x)}\right)^{1/3}$

$= \left(x^2\right)^{-2/3} x^{1/3} = x^{-4/3} x^{1/3} = x^{-3/3} = x^{-1} = \frac{1}{x}$

6. a) $\log\left(\frac{wx^2}{yz^2}\right) = \log(wx^2) - \log(yz^2)$

$= \log(w) + \log(x^2) - (\log(y) + \log(z^2))$

$= \log(w) + 2\cdot\log(x) - \log(y) - 2\log(z)$

b) $\ln\left(\frac{a^2}{a^{2/3}}\right) = \ln\left(a^{\frac{6}{3}-\frac{2}{3}}\right) = \ln\left(a^{4/3}\right)$

$= \frac{4}{3}\cdot\ln(a)$

7. a) $\log(x) + \log(y) - \log(z) = \log(xy) - \log(z)$

$= \log\left(\frac{xy}{z}\right)$

b) $3\cdot\ln x - 3\cdot\ln y - 6\ln z$

$= 3(\ln x - \ln y - 2\ln z)$

$= 3(\ln x - (\ln y + \ln z^2))$

$= 3(\ln x - \ln(yz^2))$

$= 3\left(\ln\frac{x}{yz^2}\right) = \ln\left(\left(\frac{x}{yz^2}\right)^3\right)$

$= \ln\left(\frac{x^3}{y^3z^6}\right)$