TBM 7B, 9.9.15

1 a) $3^x = 2^{x+1}$

$3^x = 2^1 \cdot 2^x$

$\frac{3^x}{2^x} = 2$

$\left(\frac{3}{2}\right)^x = 2 \quad / \log_2$

$x \cdot \log_2\left(\frac{3}{2}\right) = \log_2 2$

$x\left(\log_2 3 - \log_2 2\right) = 1$

$x = \frac{1}{\log_2 3 - 1}$

oder:

$x \cdot \ln\left(3/2\right) = \ln 2$

$x = \frac{\ln 2}{\ln(3/2)}$

b) $2^{3x} = 3^{2x}$

$3x \cdot \ln 2 = 2x \cdot \ln 3$

$x(3\ln 2 - 2\ln 3) = 0 \quad / :(\ldots)$

$x = 0$

oder:

klar, dass Gleichung nur gelten kann, wenn Exponent $= 0$

$(a^0 = 1)$

$\Rightarrow 2x = 3x = 0$

$\Rightarrow x = 0$

c) $6^{\sqrt{x}} = 3^x$

$\sqrt{x} \cdot \ln 6 = x \cdot \ln 3$

$\rightarrow x = 0$ ist Lösung!

$x \neq 0: \quad \sqrt{x} \cdot \ln 6 = x \cdot \ln 3 \quad / :\sqrt{x}$

$\ln 6 = \frac{x}{\sqrt{x}} \ln 3$

$\ln 6 = \sqrt{x} \cdot \ln 3$

$\left(\frac{\ln 6}{\ln 3}\right)^2 = x$

$\mathbb{L} = \left\{0; \left(\frac{\ln 6}{\ln 3}\right)^2\right\}$

d)

$2^{x+1} = 3^x + 3^{x+1}$

$2 \cdot 2^x = 3^x + 3 \cdot 3^x$

$2 \cdot 2^x = 3^x(1+3)$

$2 \cdot 2^x = 4 \cdot 3^x$

$2^x = 2 \cdot 3^x$

$\frac{2^x}{3^x} = 2$

$\left(\frac{2}{3}\right)^x = 2$

$x = \frac{\ln 2}{\ln(2/3)}$

e) $e^x + e^{x+1} + e^{x+2} = 2$

$e^x + e \cdot e^x + e^2 e^x = 2$

$e^x(1+e+e^2) = 2$

$e^x = \frac{2}{1+e+e^2}$

$x = \ln\left(\frac{2}{1+e+e^2}\right)$

f) $a^x = b^{x+n}$

$a^x = b^n \cdot b^x$

$\left(\frac{a}{b}\right)^x = b^n$

$x \cdot \ln(a/b) = n \cdot \ln b$

$x = \frac{n \cdot \ln b}{\ln(a/b)}$

(2)

a) $\ln x = 3 \;/ e^x$

$x = e^3$

b) $\ln(x+2) = 1 \;/ e^x$

$x+2 = e$

$x = e-2$

c) $\log(3x-2) = 2$

$3x - 2 = 10^2$

$3x = ~~98~~\ 102$

$\underline{\underline{x = 34}}$

d) $\log(2-x) = -2$

$2 - x = 10^{-2}$

$x = 2 - 10^{-2}$

$= 2 - 0.01 = \underline{\underline{1.99}}$

e) $\log x + \log(x-3) = 1$

$\log\left(x(x-3)\right) = 1$

$x(x-3) = 10$

$x^2 - 3x - 10 = 0$

$(x+2)(x-5) = 0$

$x_1 = -2$

$x_2 = 5$

f) $\ln(3-x) = 2 - \ln(x-5)$

$\ln(3-x) + \ln(x-5) = 2$

$\ln\left((3-x)(x-5)\right) = 2$

$(3-x)(x-5) = e^2$

$-x^2 + 8x - 15 - e^2 = 0$

$x^2 - 8x + 15 + e^2 = 0$

↳ QGL, Lösungsformel für quad. Gl. verwenden

③

a) $10^{x^2+3x} = \frac{1}{100}$ / log

$x^2 + 3x = -2$

$x^2 + 3x + 2 = 0$

$(x+1)(x+2) = 0$

$x_1 = -1$

$x_2 = -2$

b) $\ln x - \ln(x+1) = \ln 7 - \ln 8$

$\ln\left(\frac{x}{x+1}\right) = \ln\left(\frac{7}{8}\right)$ / e^x

$\frac{x}{x+1} = \frac{7}{8}$

$8x = 7x + 7$

$x = 7$

c) ~~$1 + 7^1 + 7^x + 7^{x+1} + 7^{x+2}$~~

$1 + 2 + 2^x + 2^{x+1} + 2^{x+2} = 31$

$2^x + 2 \cdot 2^x + 2^2 \cdot 2^x = 28$

$2^x(1 + 2 + 4) = 28$

$2^x = 4$

$x = 2$

d) $\ln(x) = \ln(5) - \ln(x+4)$

$\ln(x) + \ln(x+4) = \ln(5)$

~~$\ln\left(\frac{x}{x+4}\right) = \ln(5)$ / e^x~~

~~$\frac{x}{x+4} = 5$~~

~~$x = 5x + 20$~~

~~$-20 = 4x$~~

~~$x = -5$~~

$\ln(x(x+4)) = \ln 5$

$x^2 + 4x - 5 = 0$

$(x-1)(x+5) = 0$

$x_1 = -5$

$x_2 = 1$

(4) a) $\log_a x = \frac{\ln x}{\ln a}$

$$\frac{\log_a x}{\log_a y} = \frac{\frac{\ln x}{\ln a}}{\frac{\ln y}{\ln a}} = \frac{\ln x}{\ln a}$$

analog für rechte Seite

b) $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$= -(\ln y - \ln x)$

$= -\left(\ln\left(\frac{y}{x}\right)\right)$

$= -\ln\left(\frac{y}{x}\right)$

(5) $\log_2 a = \frac{\ln a}{\ln 2}$

$X = \log_2 a \iff 2^x = a \quad / \ln$

$x \cdot \ln 2 = \ln a$

$x = \frac{\ln a}{\ln 2}$

(6)

a) $\ln(x-3e) = 1 / e$ → $\ln e = 1$

$x - 3e = e^1$ also muss $x-3e = e$

$x = 4e$ sein

b) $10^x + 10^{x+3} = 100.1$

$10^x + 10^3 \cdot 10^x = 100.1$

$10^x(1 + 10^3) = 100.1$

$10^x = \frac{100.1}{1001} = \frac{1}{10} = 10^{-1}$

$\underline{\underline{x = -1}}$

c) $\log x = 2 - \log(x-21)$

$\log x + \log(x-21) = 2$

$\log(x(x-21)) = 2$

$x^2 - 21x = 100$

$x^2 - 21x - 100 = 0$

$(x+4)(x-25) = 0$ $x_1 = -4$ / $x_2 = 25$

d) $\log_2(x-1) + \log_2(x+2) = 2$

$\log_2((x-1)(x+2)) = 2$

$x^2 + x - 2 = 2^2$

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$ $x_1 = 2$ / $x_2 = -3$

e) $3^{x+1} + 3^{x} = 36$

$3 \cdot 3^{x} + 3^{x} = 36$

$3^{x}(3+1) = 36$

$3^{x} = 9$

$x = 2$

f) $5 \cdot 3^{x+1} = 6 \cdot 5^{x} - 5 \cdot 3^{x-1}$

$5 \cdot 3^{x+1} + 5 \cdot 3^{x-1} = 6 \cdot 5^{x}$

$5 \cdot 3^{1} \cdot 3^{x} + 5 \cdot 3^{-1} \cdot 3^{x} = 6 \cdot 5^{x}$

$3^{x}\left(15 + \frac{5}{3}\right) = 6 \cdot 5^{x}$

$$\frac{\frac{50}{3}}{6} = \frac{5^{x}}{3^{x}}$$

$$\frac{50}{18} = \left(\frac{5}{3}\right)^{x}$$

$$x = \frac{\ln(50/18)}{\ln(5/3)} = \frac{\ln(25/9)}{\ln(5/3)} = \frac{\ln(9/25)}{\ln(3/5)}$$

g) $2^{x+1} + 2^{x+2} = 48$

$2 \cdot 2^{x} + 4 \cdot 2^{x} = 48$

$6 \cdot 2^{x} = 48$

$2^{x} = 8$

$x = 3$

(7)

$$\log(x) - \log_3(x) = 2\cdot\log(3) - 2$$

$$\log(x) - \frac{\log(x)}{\log(3)} = 2\cdot\log(3) - 2$$

$$\left(1 - \frac{1}{\log 3}\right)\left(\log(x)\right) = 2\cdot\log(3) - 2$$

$$\log(x) = \frac{2\cdot\log(3) - 2}{1 - \frac{1}{\log(3)}}$$

$$= \frac{2\left(\log(3) - 1\right)}{\frac{\log(3) - 1}{\log 3}} = 2\cdot\log(3)$$

$$\log(x) = 2\log(3) = \log 3^2$$

$$\underline{\underline{x = 3^2 = 9}}$$