TBM 7B, 16.9.2015

(1)

a) $\log_2\left(x-\frac{1}{3}\right) = -2 \quad / 2^x$

$$x - \frac{1}{3} = 2^{-2} = \frac{1}{4}$$

$$x = \frac{1}{4} + \frac{1}{3} = \underline{\underline{\frac{7}{12}}}$$

b) $3^{x+2} = 9^x \quad / \ln$ (oder beliebiger log)

$$(x+2)\cdot \ln 3 = x \cdot \ln 9$$

$$= x \cdot \ln 3^2$$

$$= x \cdot 2 \cdot \ln 3$$

$$x \cdot \ln 3 + 2 \cdot \ln 3 = 2x \cdot \ln 3 \quad / - x \cdot \ln 3$$

$$2 \cdot \ln 3 = x \cdot \ln 3 \quad / : \ln 3$$

$$\underline{\underline{x = 2}}$$

c) $10^{x-1} + 10^x + 10^{x+1} = 11'100$

$$10^{-1} \cdot 10^x + 10^x + 10 \cdot 10^x = 11'100$$

$$10^x\left(10^{-1} + 1 + 10\right) = 11'100$$

$$10^x \cdot 11.1 = 11'100$$

$$10^x = \frac{11'100}{11.1}$$

$$10^x = 1000$$

$$\underline{\underline{x = 3}}$$

d) $\ln(x+3) = \ln(3) - \ln(x+5)$

$\ln(x+3) + \ln(x+5) = \ln(3)$

$\ln\left((x+3)(x+5)\right) = \ln(3) \quad / e^{x}$

$x^2 + 8x + 15 = 3$

$x^2 + 8x + 12 = 0$

$(x+2)(x+6) = 0 \quad x_1 = -2$

$\mathbb{L} = \{-2\}$

$x_2 = -6$

keine Lösung!

$\underbrace{\ln(x+3)}_{>0}$

e) $2 \cdot 2^{x+1} + 2^{x-1} = 4 \cdot 3^{x-1}$

$2 \cdot 2 \cdot 2^x + 2^{-1} 2^x = 4 \cdot 3^{-1} \cdot 3^x$

$4 \cdot 2^x + \frac{1}{2} \cdot 2^x = 4 \cdot \frac{1}{3} \cdot 3^x$

$\frac{9}{2} \cdot 2^x = \frac{4}{3} \cdot 3^x \quad / : 3^x : \frac{9}{2}$

$\frac{2^x}{3^x} = \frac{\frac{4}{3}}{\frac{9}{2}} = \frac{8}{27} = \left(\frac{2}{3}\right)^3$

$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^3$

$\underline{\underline{x = 3}}$

f) $\log(7) - \log(x-5) - \log(x+1) = 0$

$\log(7) - \left(\log(x-5) + \log(x+1)\right) = 0$

$\log(7) - \log\left((x-5)(x+1)\right) = 0$

$\log \dfrac{7}{(x-5)(x+1)} = 0 \quad / 10^x$

$\dfrac{7}{(x-5)(x+1)} = 1 \quad / \cdot (\ )(\ )$

$7 = (x-5)(x+1)$

$7 = x^2 - 4x - 5$

$x^2 - 4x - 12 = 0$

$(x-2)(x-6) = 0$

$x_1 = 2 \ ; \ x_2 = 6$

$\log(x-5)$ nicht

def. für $x_1 = 2$

$\mathbb{L} = \{6\}$

$\log(7) = \log(x-5) + \log(x+1)$

$\log(7) = \log\left((x-5)(x+1)\right) \quad / 10^x$

$7 = (x-5)(x+1)$

⋮

g) $\log(x^2) + \log(\sqrt{x}) = -5$

$2 \cdot \log x + \log(x^{1/2}) = -5$

$2 \cdot \log x + \frac{1}{2} \cdot \log x = -5$

$\frac{5}{2} \cdot \log x = -5 \quad / \cdot \frac{2}{5}$

$\log x = -2 \quad / 10^x$

$\underline{\underline{x = 10^{-2} = \frac{1}{100} = 0.01}}$

h) $\log(x) + \log_2(x) + \log_5(x) = \frac{5}{\log_2(10)} + 5 + \frac{5}{\log_2(5)}$

$\rightarrow$ alle Logarithmen in Zer-Log umwandeln:

$$\frac{\log_2(x)}{\log_2(10)} + \log_2(x) + \frac{\log_2(x)}{\log_2(5)} = 5\left(\frac{1}{\log_2(10)} + 1 + \frac{1}{\log_2(5)}\right)$$

$$\log_2(x) \cdot \left(\frac{1}{\log_2(10)} + 1 + \frac{1}{\log_2(5)}\right) = 5\left(\frac{1}{\log_2(10)} + 1 + \frac{1}{\log_2(5)}\right)$$

$$\log_2(x) = 5 \quad / 2^{\wedge}$$

$$\underline{\underline{x = 2^5 = 32}}$$

Etwas aufwendiger wird's, wenn 10er-Log verwendet wird:

$$\log(x) + \log_2(x) + \log_5(x) = 5\left(\frac{1}{\log_2(10)} + 1 + \frac{1}{\log_2(5)}\right)$$

$$\log(x) + \frac{\log(x)}{\log(2)} + \frac{\log(x)}{\log(5)} = 5\left(\frac{1}{\frac{\log(10)}{\log 2}} + 1 + \frac{1}{\frac{\log(5)}{\log(2)}}\right)$$

$$\log(x)\left(1 + \frac{1}{\log(2)} + \frac{1}{\log 5}\right) = 5\left(\frac{\log 2}{\underbrace{\log 10}_{\log 10 = 1}} + \frac{\log 2}{\log 2} + \frac{\log 2}{\log 5}\right)$$

$$\log(x)\left(1 + \frac{1}{\log(2)} + \frac{1}{\log(5)}\right) = 5 \cdot \log(2) \cdot \left(1 + \frac{1}{\log(2)} + \frac{1}{\log(5)}\right)$$

$$\log(x) = 5 \cdot \log(2) \quad / 10^{\wedge}$$

$$x = 10^{5 \cdot \log(2)}$$

$$= \left(10^{\log(2)}\right)^5 = 2^5 = \underline{\underline{32}}$$

(2)

Feld	# Reiskörner	Σ	
1	1	1	$2^1 - 1$
2	2	3	$2^2 - 1$
3	4	7	$2^3 - 1$
4	8	15	
5	16	31	
6	32	63	
		127	
		255	

$$\text{Summe} = 2^n - 1$$

$$\frac{2^n - 1}{50 \cdot 1000 \cdot 8 \cdot 10^9} = 1$$

$$2^n = 4 \cdot 10^{14} - 1 \quad / \ln$$

$$n \cdot \ln(2) = \ln(4 \cdot 10^{14} - 1)$$

$$n = \frac{\ln(4 \cdot 10^{14} - 1)}{\ln(2)} \approx 48.51$$

$\longrightarrow$ 49. Feld