3HD3, 8.12.2017

① a) $f(x) = x^2 - 4x + 8$

$= x^2 - 4x + 4 - 4 + 8$

$= (x-2)^2 - 4 + 8$

$f(x) = (x-2)^2 + 4 \qquad S(2/4)$

$a = 1 \;/\; b = -4 \;/\; c = 8$

$S\left(-\frac{b}{2a} \,/\, c - \frac{b^2}{4a}\right) = S\left(-\frac{-4}{2} \,/\, 8 - \frac{(-4)^2}{4 \cdot 1}\right)$

$= S(2/4)$

b) $g(x) = -\frac{1}{3}x^2 - 2x + 3$

$= -\frac{1}{3}(x^2 + 6x) + 3$

$= -\frac{1}{3}(x^2 + 6x + 9 - 9) + 3$

$= -\frac{1}{3}\left[(x+3)^2 - 9\right] + 3$

$= -\frac{1}{3}(x+3)^2 + 3 + 3$

$g(x) = -\frac{1}{3}(x+3)^2 + 6 \qquad S(-3/6)$

$a = -\frac{1}{3} \;/\; b = -2 \;/\; c = 3$

$S\left(-\frac{-2}{2 \cdot (-\frac{1}{3})} \,/\, 3 - \frac{(-2)^2}{4 \cdot (-\frac{1}{3})}\right)$

$S\left(\frac{2}{-\frac{2}{3}} \,/\, 3 - \frac{4}{-\frac{4}{3}}\right) = S\left(-2 \cdot \frac{3}{2} \,/\, 3 + 4 \cdot \frac{3}{4}\right)$

$S(-3/6)$

c) $h(x) = \frac{1}{5}x^2 - \frac{6}{5}x + \frac{24}{5}$

$= \frac{1}{5}(x^2 - 6x) + \frac{24}{5}$

$= \frac{1}{5}(x^2 - 6x + 9 - 9) + \frac{24}{5}$

$= \frac{1}{5}[(x-3)^2 - 9] + \frac{24}{5}$

$= \frac{1}{5}(x-3)^2 - \frac{9}{5} + \frac{24}{5}$

$h(x) = \frac{1}{5}(x-3)^2 + 3 \qquad S(3/3)$

$a = \frac{1}{5} \;/\; b = -\frac{6}{5} \;/\; c = \frac{24}{5}$

$S\left(-\frac{b}{2a} \,/\, c - \frac{b^2}{4a}\right) = S\left(-\frac{-\frac{6}{5}}{2\cdot\frac{1}{5}} \,/\, \frac{24}{5} - \frac{(-\frac{6}{5})^2}{4\cdot\frac{1}{5}}\right)$

$= S\left(\frac{\frac{6}{5}}{\frac{2}{5}} \,/\, \frac{24}{5} - \frac{\frac{36}{25}}{\frac{4}{5}}\right) = S\left(\frac{6}{5}\cdot\frac{5}{2} \,/\, \frac{24}{5} - \frac{36}{25}\cdot\frac{5}{4}\right)$

$= S\left(3 \,/\, \frac{24}{5} - \frac{9}{5}\right) = S(3/3)$

d) $k(x) = -2x^2 + 16x - 30$

$= -2(x^2 - 8x) - 30$

$= -2(x^2 - 8x + 16 - 16) - 30$

$= -2[(x-4)^2 - 16] - 30$

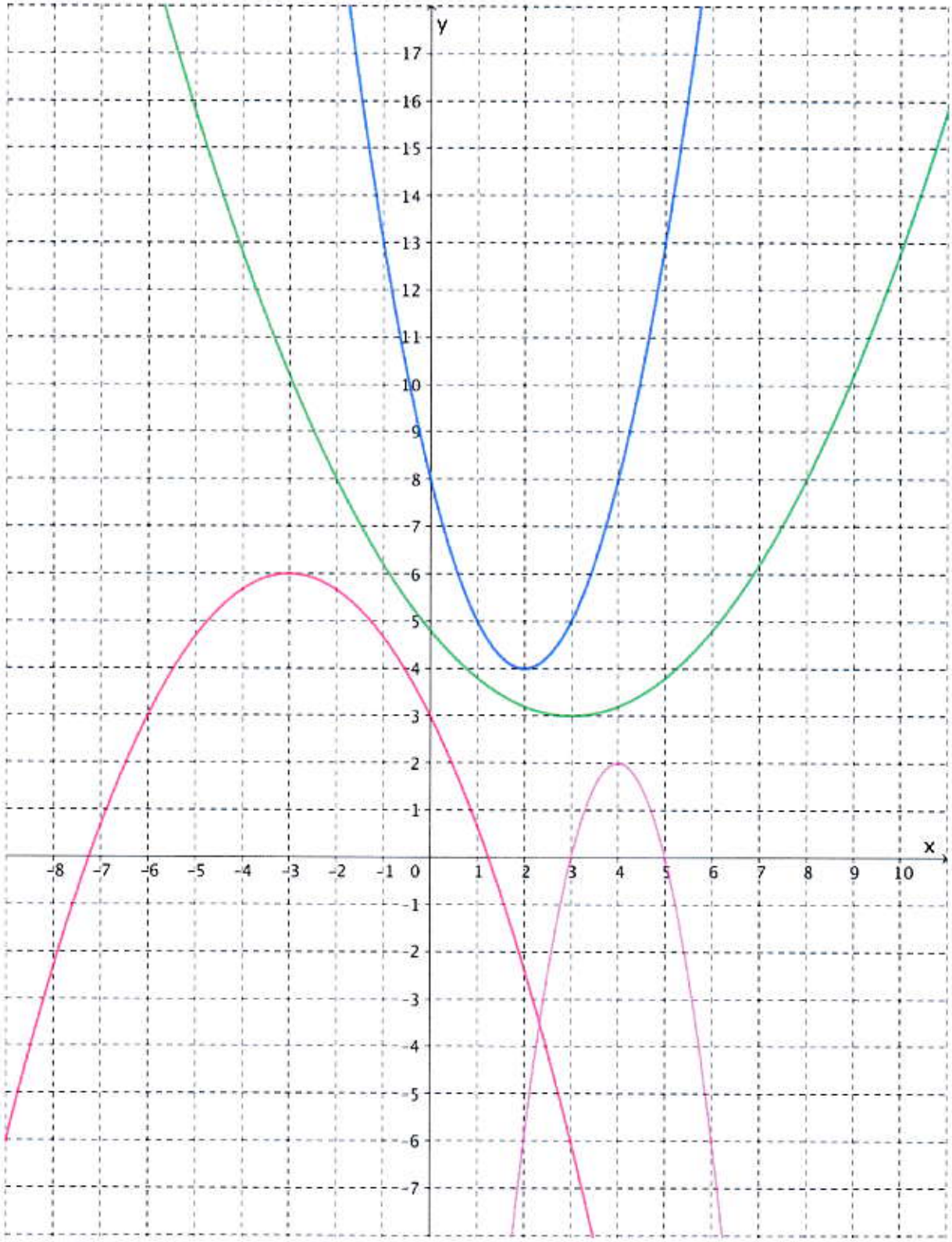
$= -2(x-4)^2 + 32 - 30$

$k(x) = -2(x-4)^2 + 2 \qquad S(4/2)$

$S\left(-\frac{b}{2a} \,/\, c - \frac{b^2}{4a}\right) \qquad a = -2 \;/\; b = 16 \;/\; c = -30$

$S\left(-\frac{16}{2(-2)} \,/\, -30 - \frac{16^2}{4\cdot(-2)}\right) = S\left(4 \,/\, -30 + \frac{256}{8}\right)$

$S(4/-30+32) = S(4/2)$


y
x
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

(2) $y = ax^2 + bx + c$

A(1/5): $5 = a \cdot 1^2 + b \cdot 1 + c$

$5 = a + b + c$ (I.)

B(3/7): $7 = a \cdot 3^2 + b \cdot 3 + c$

$7 = 9a + 3b + c$ (II.)

C(-1/7) $7 = a(-1)^2 + b(-1) + c$

$7 = a - b + c$ (III.)

} 1,5

II. − I.: $2 = 8a + 2b \quad | : 2$

$1 = 4a + b$

II. − III.: $0 = 8a + 4b \quad | : 4$

$0 = 2a + b$

} 1

$1 = 2a$

$a = \frac{1}{2}$

} 1/2

$0 = 2a + b \quad , a = \frac{1}{2}$

$0 = 2 \cdot \frac{1}{2} + b$

$0 = 1 + b$

$b = -1$

} 1/2

$5 = a + b + c$

$5 = \frac{1}{2} - 1 + c$

$c = \frac{11}{2}$ } 1/2

$p(x) = \frac{1}{2}x^2 - x + \frac{11}{2}$

(3) $S(-2/-1)$, $P(-4/7)$

$f(x) = a(x-s)^2 + h$

$f(x) = a(x+2)^2 - 1$ (2)

$P(-4/7)$: $7 = a(-4+2)^2 - 1$

$7 = 4a - 1 \quad /+1$

$8 = 4a$

$a = 2$ (1)

$\hookrightarrow f(x) = 2(x+2)^2 - 1$

$= 2(x^2+4x+4) - 1$

$\underline{f(x) = 2x^2 + 8x + 7}$ (1)

(4) p_1: $S(4/10) \Rightarrow p_1(x) = a(x-4)^2 + 10$

$a = -\frac{1}{2} \Rightarrow p_1(x) = -\frac{1}{2}(x-4)^2 + 10$

$p_1(x) = -\frac{1}{2}x^2 + 4x + 2$

p_2: $S(-6/-4) \Rightarrow p_2(x) = a(x+6)^2 - 4$

$a = 2 \Rightarrow p_2(x) = 2(x+6)^2 - 4$

$p_2(x) = 2x^2 + 24x + 68$

p_3: $S(2/2) \Rightarrow p_3(x) = a(x-2)^2 + 2$

$a = \frac{1}{5} \Rightarrow p_3(x) = \frac{1}{5}(x-2)^2 + 2$

$p_3(x) = \frac{1}{5}x^2 - \frac{4}{5}x + \frac{14}{5}$

p_4: $S(5/1) \Rightarrow p_4(x) = a(x-5)^2 + 1$

$a = -\frac{1}{3} \Rightarrow p_4(x) = -\frac{1}{3}(x-5)^2 + 1$

~~$p_4(x) = -\frac{1}{4}x^2 + \frac{5}{2}x - \frac{21}{4}$~~

$p_4(x) = -\frac{1}{3}(x-5)^2 + 1$

$p_4(x) = -\frac{1}{3}x^2 + \frac{10}{3}x - \frac{22}{3}$

(5)

$$g(x) = p(x)$$

$$-3x - 8 = -\frac{1}{2}x^2 - 2x + 4 \quad | \cdot (-2)$$

$$6x + 16 = x^2 + 4x - 8 \quad | -6x - 16$$

$$0 = x^2 - 2x - 24 \qquad 2\,P.$$

$$0 = (x+4)(x-6) \qquad 3\,P.$$

$x_1 = -4 \quad y_1 = 4 \qquad 1\,P. \qquad S_1(-4/4)$

$x_2 = 6 \quad y_2 = -26 \qquad S_2(6/-26)$

(6)

$$p_1(x) = p_2(x)$$

$$x^2 + 2x - 1 = -\frac{1}{3}x^2 + \frac{22}{3}x - 5 \quad | \cdot 3$$

$$3x^2 + 6x - 3 = -x^2 + 22x - 15$$

$$4x^2 - 16x + 12 = 0 \quad | : 4$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$x_1 = 1 \quad y_1 = 2 \qquad S_1(1/2)$

$x_2 = 3 \quad y_2 = 14 \qquad S_2(3/14)$