TBM3, 3.11.2015

Trigo II

① $a = 4m$

$b = 5m$

$c = 6m$

$\alpha = 41.41°$

$\beta = 55.77°$

$\gamma = 82.82°$

$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$

$$\gamma = \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right)$$

$\gamma = 82.8192°$ (41.4096221...)

$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$

$$\alpha = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$$

$\alpha = 41.4096°$ (55.7711337...)

$b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$

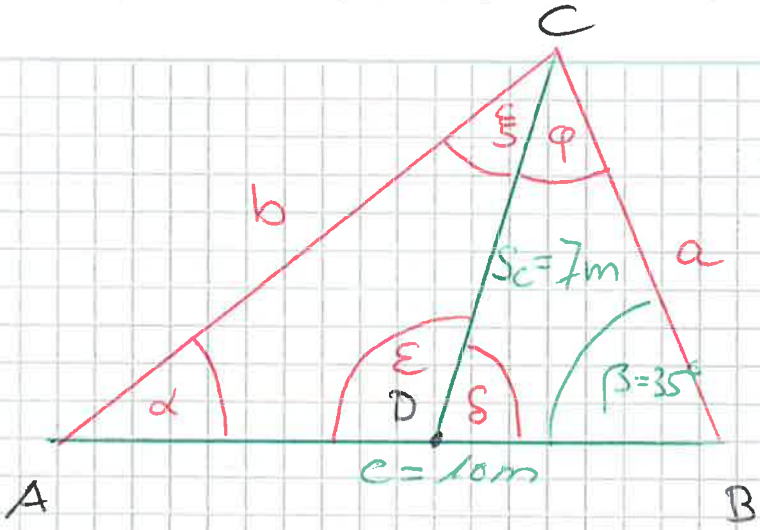
$$\beta = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right)$$

$\beta = 55.7711°$ (82.8192442)

$\cancel{\gamma = 180° - \alpha - \beta}$

$\cancel{\gamma = 68.24°}$ ~~(68.2374878)~~

(2)



Winkel φ: $\frac{\sin\varphi}{c/2} = \frac{\sin\beta}{s_c} \Rightarrow \varphi = \sin^{-1}\left(\frac{c \cdot \sin\beta}{2 s_c}\right)$

$\underline{\underline{\varphi \cong 24.19°}}$ (24.1858368..)

$\delta = 180° - \beta - \varphi \cong 120.81°$

$\underline{\underline{\delta \cong 120.81°}}$ (120.814169)

Seite a: $\frac{a}{\sin\delta} = \frac{s_c}{\sin\beta} \Rightarrow a = \frac{s_c \cdot \sin\delta}{\sin\beta}$

$\underline{\underline{a \cong 10.48m}}$ (10.4813105)

Seite b: $b = \sqrt{a^2 + c^2 - 2ac \cdot \cos\beta}$

$\underline{\underline{b = 6.18m}}$ (6.1759316)

Winkel α: $a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$

$\alpha = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$

$\underline{\underline{\alpha \cong 76.76°}}$ (76.7625122)

$\gamma = 180° - \alpha - \beta$

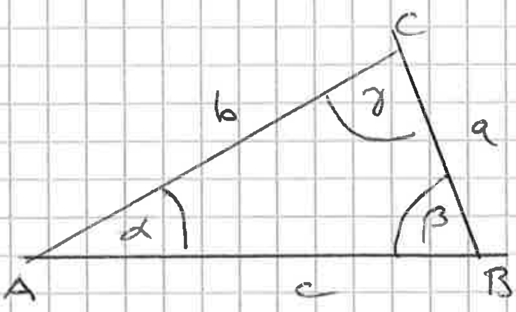
$\gamma = 68.24°$ (68.2374878)

TBM 3, 3. 11. 2015

(1) $a = 4m$

$b = 5m$

$c = 6m$



$$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$$

$$\frac{c^2 - a^2 - b^2}{-2ab} = \cos\gamma$$

$$\cos^{-1}\left(\frac{c^2-a^2-b^2}{-2ab}\right) = \gamma \cong 82.819'244\ldots$$

$$\cong \underline{\underline{82.82^\circ}}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$$

$$\frac{a^2 - b^2 - c^2}{-2bc} = \cos\alpha$$

$$\cos^{-1}\left(\frac{a^2-b^2-c^2}{-2bc}\right) = \alpha \cong 41.409'622\ldots$$

$$\cong \underline{\underline{41.41^\circ}}$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$$

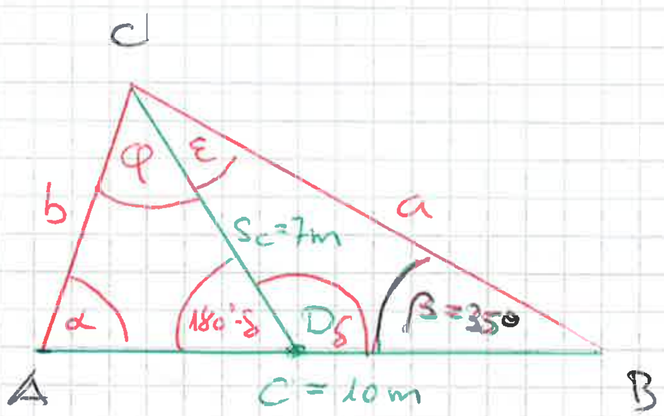
$$\cos^{-1}\left(\frac{b^2-a^2-c^2}{-2ac}\right) = \beta \cong 55.771'137\ldots$$

$$\cong \underline{\underline{55.77^\circ}}$$

②

$\triangle DBC$:

$$\frac{\sin\varepsilon}{c/2} = \frac{\sin\beta}{s_c}$$



$$\sin\varepsilon = \frac{5\cdot\sin 35^\circ}{7}$$

$$\varepsilon = \sin^{-1}\left(\frac{5}{7}\cdot\sin 35^\circ\right) \approx 24.185'830'1\ldots$$

$$\approx \underline{24.19^\circ}$$

$$\delta = 180^\circ - 35^\circ - \varepsilon \approx \underline{120.81^\circ}$$

$$\frac{a}{\sin\delta} = \frac{s_c}{\sin 35^\circ}$$

$$a = \frac{s_c\cdot\sin\delta}{\sin 35^\circ} \approx 10.481'310\ldots \approx \underline{\underline{10.48m}}$$

$$b = \sqrt{\left(\frac{c}{2}\right)^2 + s_c^2 - 2\cdot\frac{c}{2}\cdot s_c\cdot\cos(180^\circ-\delta)}$$

$$= \sqrt{5^2+7^2-2\cdot 5\cdot 7\cdot\cos(180^\circ-\delta)}$$

$$\approx 6.175'931'6\ldots \approx \underline{\underline{6.18\,m}}$$

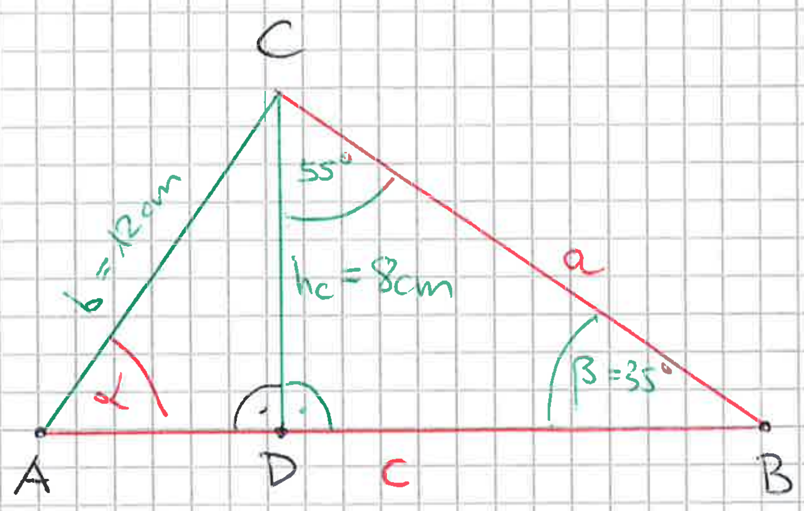
$$\frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \Rightarrow \gamma = \sin^{-1}\left(\frac{c}{b}\cdot\sin\beta\right)$$

$$\approx 68.237'487'8\ldots$$

$$\approx \underline{\underline{68.24^\circ}}$$

$$\alpha = 180^\circ - \beta - \gamma \approx 76.762'512\ldots \approx \underline{\underline{76.76^\circ}}$$

3



$$\tan\beta = \frac{h_c}{\overline{DB}} \Rightarrow \overline{DB} = \frac{h_c}{\tan\beta} \cong 11.425'184$$

$$\cong \underline{11.43cm}$$

$$\sin\alpha = \frac{h_c}{b} \Rightarrow \alpha = \sin^{-1}\left(\frac{8}{12}\right) = \sin^{-1}\left(\frac{2}{3}\right)$$

$$\cong 41.810'314'87...$$

$$\alpha \cong \underline{\underline{41.81°}}$$

$$\tan\alpha = \frac{h_c}{\overline{AD}} \Rightarrow \overline{AD} = \frac{h_c}{\tan\alpha} \cong 8.944'271'91$$

$$\cong \underline{8.94cm}$$

$$c = \overline{AD} + \overline{DB} \cong 20.369'455'963...$$

$$\underline{\underline{c \cong 20.37cm}}$$

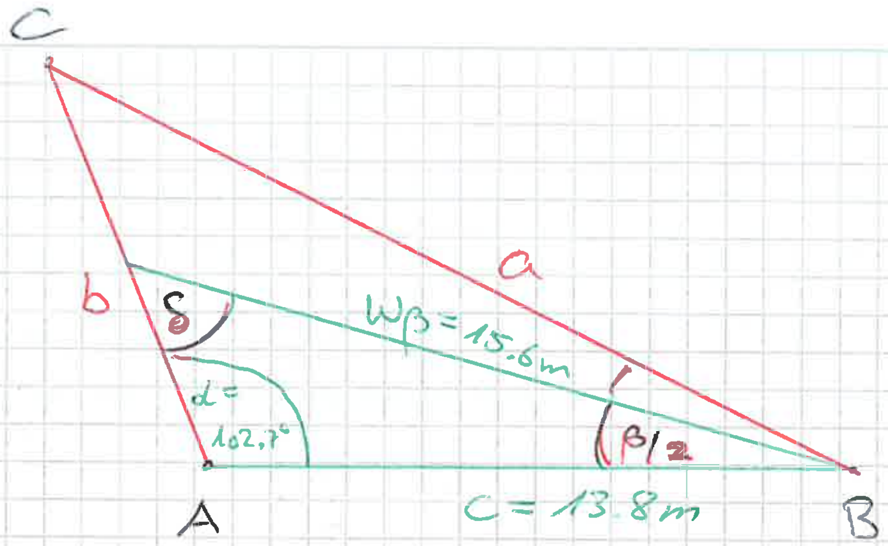
$$a = \sqrt{b^2 + c^2 - 2bc \cdot \cos\alpha} \cong \cancel{8.413'866'478}$$

$$a = 13.947'955'86... \cong \underline{\underline{13.95cm}} \qquad \cong \underline{\underline{\cancel{8.41cm}}}$$

$$\gamma = 180° - \alpha - \beta \cong 103.187'685'104...$$

$$\underline{\underline{\gamma \cong 103.19°}}$$

④



$$\frac{\sin\delta}{c} = \frac{\sin\alpha}{w_\beta} \Rightarrow \delta = \sin^{-1}\left(\frac{c \cdot \sin\alpha}{w_\beta}\right)$$

$$\delta \cong 59.652'032'908...$$

$$\underline{\delta \cong 59.65°}$$

$$\frac{\beta}{2} = 180° - \alpha - \delta \cong 17.647'967...$$

$$\beta \cong 35.295'934... \cong \underline{\underline{35.3°}}$$

$$\gamma = 180° - \alpha - \beta \cong 42.004'065'817...$$

$$\underline{\underline{\gamma \cong 42°}}$$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow b = \frac{c \cdot \sin\beta}{\sin\gamma} \cong 11.915'472'93...$$

$$\cong \underline{\underline{11.92m}}$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = \frac{b \cdot \sin\alpha}{\sin\beta} \cong 20.117'620'85...$$

$$\cong \underline{\underline{20.12m}}$$

(5)

$$\sin\alpha = \frac{\overline{QM}}{\overline{MR}} \quad ; \quad \overline{MR} = \overline{MP} = \text{Radius}$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{10}{25}\right) \cong 23.578'178'478\ldots$$

$$\underline{\underline{\alpha \cong 23.58°}}$$

$$\tan\alpha = \frac{\overline{MQ}}{\overline{QR}} \Rightarrow \overline{QR} = \frac{\overline{MQ}}{\tan\alpha}$$

$$\overline{QR} \cong 22.912'878'474\ldots$$

$$\underline{\underline{\overline{QR} \cong 22.91\,cm}}$$

Fläche = Sektor(MPR) + ΔMRQ
− Viertelkreis

$$A_{Sektor(MPR)} = \frac{\pi \cdot 25^2}{360°} \cdot \alpha \cong 128.599'014'396\ldots$$

$$\underline{\underline{\cong 128.6\,cm^2}}$$

$$A_{\Delta MRQ} = \frac{1}{2}\,\overline{MQ} \cdot \overline{QR} = 114.564'392'374$$

$$\underline{\underline{\cong 114.56\,cm^2}}$$

$$A_{Viertelkreis} = \frac{1}{4}\pi \cdot 10^2 \cong 78.539'816'339\ldots$$

$$\cong 78.54\,cm^2$$

rote Fläche = 164.623'590'47… = $\underline{\underline{164.62\,cm^2}}$