TBM 6A, 28.2.2014

① $\vec{r}_A = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$; $\vec{r}_B = \begin{pmatrix} 5 \\ 4 \\ 7 \end{pmatrix}$; $g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} + s \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$g \parallel$ z-Achse, $g \neq$ z-Achse

② $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \cdot (-3) = \begin{pmatrix} 3 \\ -6 \\ -3 \end{pmatrix} \Rightarrow g \parallel h$

$g = h$? → liegt $(8|-8|-1)$ auf g ?

$$\left.\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} + s \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \\ -1 \end{pmatrix} \quad \begin{matrix} s = -5 \\ s = -5 \\ s = -5 \end{matrix} \right\} \quad \underline{\underline{g \equiv h}}$$

③ $g = h \Rightarrow s = -2,\ t = -3$

$S(-6|5|-1)$

$$\cos\alpha = \frac{\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}}{\sqrt{4^2+1^2+2^2}\sqrt{1^2+3^2}} = \frac{4+6}{\sqrt{21}\sqrt{10}} = \frac{10}{\sqrt{210}}$$

$$\cos\alpha = \frac{\sqrt{210}}{21} \Rightarrow \alpha = 46.3647°$$

$$\hat{=}\ 46.36°$$

(4) $\vec{r}_C = \begin{pmatrix} s-5 \\ s-4 \\ -s+1 \end{pmatrix}$

$\vec{CA} \cdot \vec{CB} = 0 \iff \vec{CA} \perp \vec{CB}$

$\begin{pmatrix} 11-s \\ 1-s \\ s-21 \end{pmatrix} \begin{pmatrix} 16-s \\ -s-8 \\ s+2 \end{pmatrix} = 0$

$3s^2 - 39s + 126 = 0$

$\boxed{\begin{array}{ll} s_1 = 6 & C_1(1|2|-5) \\ s_2 = 7 & C_2(2|3|-6) \end{array}}$

$A = \frac{1}{2}\overline{CA} \cdot \overline{CB} = \frac{1}{2} |\vec{CA}| \cdot |\overline{CB}|$

$=$ a) $\frac{1}{2} \left|\begin{pmatrix} 5 \\ -5 \\ -15 \end{pmatrix}\right| \left|\begin{pmatrix} 10 \\ -14 \\ 8 \end{pmatrix}\right| = 15\sqrt{110} \approx 157.321$

b) $\frac{1}{2} \left|\begin{pmatrix} 4 \\ -6 \\ -14 \end{pmatrix}\right| \left|\begin{pmatrix} 9 \\ -15 \\ 9 \end{pmatrix}\right| = 3\sqrt{2666} \approx 154.9$

(5) $S \in g$:

$\begin{pmatrix} -3 \\ 8 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} S_x \\ 10 \\ S_z \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -7 \end{pmatrix}$

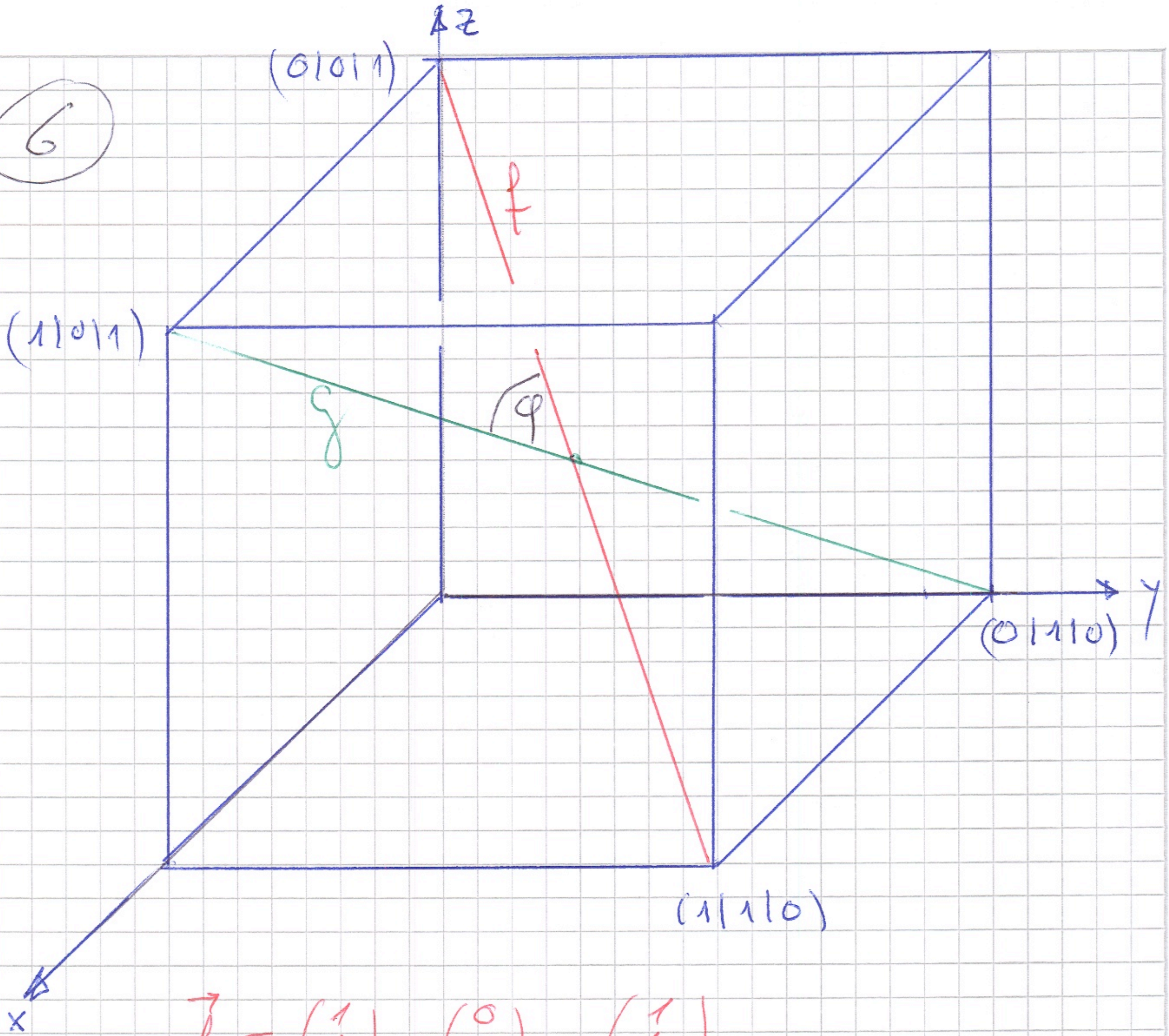
$\Rightarrow s = 2 \Rightarrow \underline{\underline{S_x = 1,\ S_z = -7}}$

$S \in h$:

$\begin{pmatrix} a \\ 5 \\ -3 \end{pmatrix} + t \begin{pmatrix} 1 \\ b \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -7 \end{pmatrix} \Rightarrow t = -1$

$\Rightarrow \underline{\underline{a = 2,\ b = -5}}$

(6)



$$\vec{f} = \begin{pmatrix}1\\1\\0\end{pmatrix} - \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\1\\-1\end{pmatrix}$$

$$\vec{g} = \begin{pmatrix}1\\0\\1\end{pmatrix} - \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$\cos\varphi = \frac{\begin{pmatrix}1\\1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\\1\end{pmatrix}}{\sqrt{3}\,\sqrt{3}} = \frac{-1}{3} = -1/3$$

$$\varphi = \arccos(-1/3) = 109.471$$

$$\varphi' = 180° - \varphi = \underline{\underline{70.529°}}$$

(7) $Q \in g \Rightarrow \vec{r}_Q = \begin{pmatrix} 2s-3 \\ s+8 \\ -4s+1 \end{pmatrix}$

$\vec{g} \cdot \vec{PQ} = 0$

$\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \begin{pmatrix} 2s-4 \\ s+1 \\ -4s-1 \end{pmatrix} = 0$

$21s - 3 = 0$

$21s = 3$

$s = 1/7 \; ; \; Q\left(-\frac{19}{7} \middle/ \frac{57}{7} \middle/ \frac{3}{7}\right)$

$|\vec{PQ}| = \frac{\sqrt{861}}{7} \cong \underline{\underline{4.19}}$