

VG TBM 6E, 17.6.2016

① a) $\vec{a} = \begin{pmatrix} -6 \\ 13.5 \end{pmatrix}; \vec{b} = \begin{pmatrix} 4 \\ -9 \end{pmatrix}$

$$\left. \begin{aligned} (-6) \cdot \left(-\frac{2}{3}\right) &= 4 \\ 13.5 \cdot \left(-\frac{2}{3}\right) &= -9 \end{aligned} \right\} \begin{array}{l} \text{kollinear} \\ \text{antiparallel} \end{array}$$

b) $\vec{a} = \begin{pmatrix} -4.2 \\ 7 \\ -6.3 \end{pmatrix}; \vec{b} = \begin{pmatrix} -1.2 \\ 2 \\ -1.8 \end{pmatrix}$

$$\left. \begin{aligned} -4.2 \cdot \frac{2}{7} &= -1.2 \\ 7 \cdot \frac{2}{7} &= 2 \\ -6.3 \cdot \frac{2}{7} &= -1.8 \end{aligned} \right\} \begin{array}{l} \text{kollinear} \\ \text{parallel} \end{array}$$

② a) $\vec{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \vec{b} = \begin{pmatrix} x \\ -5 \end{pmatrix}$

$$\begin{aligned} 2 \cdot \left(-\frac{5}{2}\right) &= -5 \\ -1 \cdot \left(-\frac{5}{2}\right) &= \frac{5}{2} \end{aligned} \quad \underline{\underline{x = \frac{5}{2}}}$$

b) $\vec{a} = \begin{pmatrix} -3 \\ 1.5 \\ 2 \end{pmatrix}; \vec{b} = \begin{pmatrix} 4 \\ y \\ -10 \end{pmatrix}$

$$(-3) \cdot \left(-\frac{4}{3}\right) = 4$$

$$\frac{3}{2} \cdot \left(-\frac{4}{3}\right) = -\frac{12}{6} = -2; \quad \underline{\underline{y = -2}}$$

$$(-10) \cdot \left(-\frac{3}{4}\right) = \frac{30}{4} = \frac{15}{2} = 7.5; \quad \underline{\underline{z = 7.5}}$$

$$\textcircled{3} \quad a) \quad \vec{v} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}; \quad v = \sqrt{3^2 + 4^2} = 5$$

$$\vec{w} = \frac{\vec{v}}{v} = \frac{1}{5} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix}$$

$$b) \quad \vec{v} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}; \quad v = \sqrt{3}$$

$$\frac{\vec{v}}{v} = \frac{1}{\sqrt{3}} \cdot \vec{v} = \frac{\sqrt{3}}{3} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/3 \\ -\sqrt{3}/3 \\ \sqrt{3}/3 \end{pmatrix}$$

$$\textcircled{4} \quad A(-13/5/-2); \quad B(-21/9/-3)$$

$$\vec{AB} = \vec{r}_B - \vec{r}_A = \begin{pmatrix} -21 \\ 9 \\ -3 \end{pmatrix} - \begin{pmatrix} -13 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$$

$$\overline{AB} = |\vec{AB}| = \sqrt{8^2 + 4^2 + 1^2} = \sqrt{81} = \underline{\underline{9}}$$

$$\textcircled{5} \quad A(-7/5/5); \quad B(-9/4/8)$$

$$\overline{AB} = 3; \quad y = z + 1$$

$$\vec{AB} = \vec{r}_B - \vec{r}_A = \begin{pmatrix} -9 \\ z+1 \\ 2 \end{pmatrix} - \begin{pmatrix} -7 \\ 5 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ z-4 \\ z-5 \end{pmatrix}$$

$$|\vec{AB}|^2 = 2^2 + (z-4)^2 + (z-5)^2 = 3^2$$

$$4 + z^2 - 8z + 16 + z^2 - 10z + 25 = 9$$

$$2z^2 - 18z + 36 = 0 \quad |:2$$

$$z^2 - 9z + 18 = 0$$

$$(z-3)(z-6) = 0$$

$$z_1 = 3$$

$$z_2 = 6$$

$$B_1(-9/4/3)$$

$$B_2(-9/7/6)$$

$$\textcircled{6} \quad M(2/3/10)$$

P auf z-Achse: $P(0/0/z)$

$$\vec{MP} = \vec{r}_P - \vec{r}_M = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 10 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ z-10 \end{pmatrix}$$

$$|\vec{MP}|^2 = 7^2$$

$$2^2 + 3^2 + (z-10)^2 = 49$$

$$4 + 9 + z^2 - 20z + 100 = 49$$

$$z^2 - 20z + 64 = 0$$

$$(z-4)(z-16) = 0$$

$$z_1 = 4$$

$$z_2 = 16$$

$$\underline{P_1(0/0/4)}$$

$$\underline{P_2(0/0/16)}$$