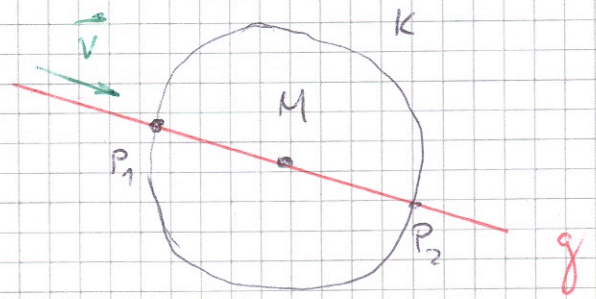
VG TBM 7A, 20.9.13

(1) $\vec{v} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$



$\vec{r}_{P_{1,2}} = \vec{r}_M \pm 35 \frac{\vec{v}}{v}$

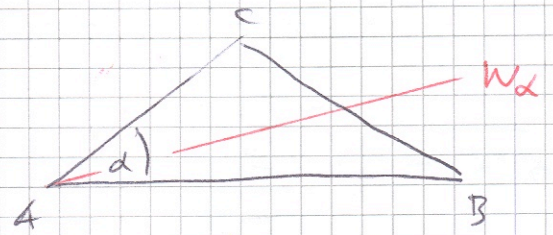
$|\vec{v}| = v = \sqrt{4^2+3^2} = 5$

$\vec{r}_{P_{1,2}} = \begin{pmatrix} 5 \\ -3 \\ -8 \end{pmatrix} \pm \frac{35}{5} \cdot \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -3 \\ -8 \end{pmatrix} \pm 7 \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$; $P_1(33/-3/13)$

$P_2(-23/-3/-29)$

(3)



Richtungsvektor der Gerade w_α:

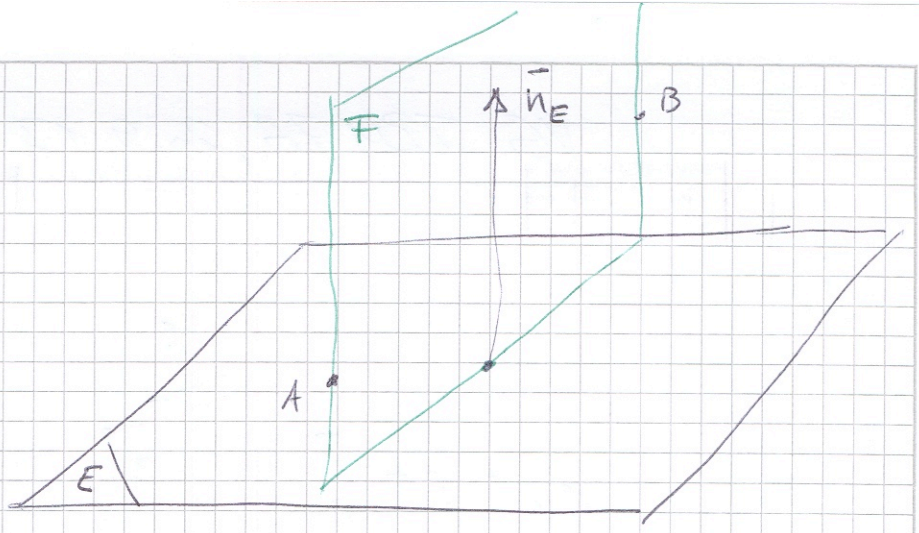
$\vec{v} = \frac{\overrightarrow{AB}}{\overline{AB}} + \frac{\overrightarrow{AC}}{\overline{AC}}$

$= \frac{1}{\sqrt{2^2+3^2+6^2}} \cdot \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} + \frac{1}{\sqrt{4^2+5^2+20^2}} \begin{pmatrix} 4 \\ 5 \\ 20 \end{pmatrix} =$

$= \frac{1}{7} \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} + \frac{1}{21} \begin{pmatrix} 4 \\ 5 \\ 20 \end{pmatrix} = \begin{pmatrix} 10/21 \\ 21/3 \\ 38/21 \end{pmatrix} \longrightarrow \begin{pmatrix} 5 \\ 7 \\ 19 \end{pmatrix}$

$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \\ -15 \end{pmatrix} + s \begin{pmatrix} 5 \\ 7 \\ 19 \end{pmatrix}$

(4)



$E: 2x - 2y + z - 8 = 0 \qquad \vec{n}_E = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

Sowohl $\vec{n}_E$ als auch $\vec{AB}$ sind parallel zur gesuchten Ebene F

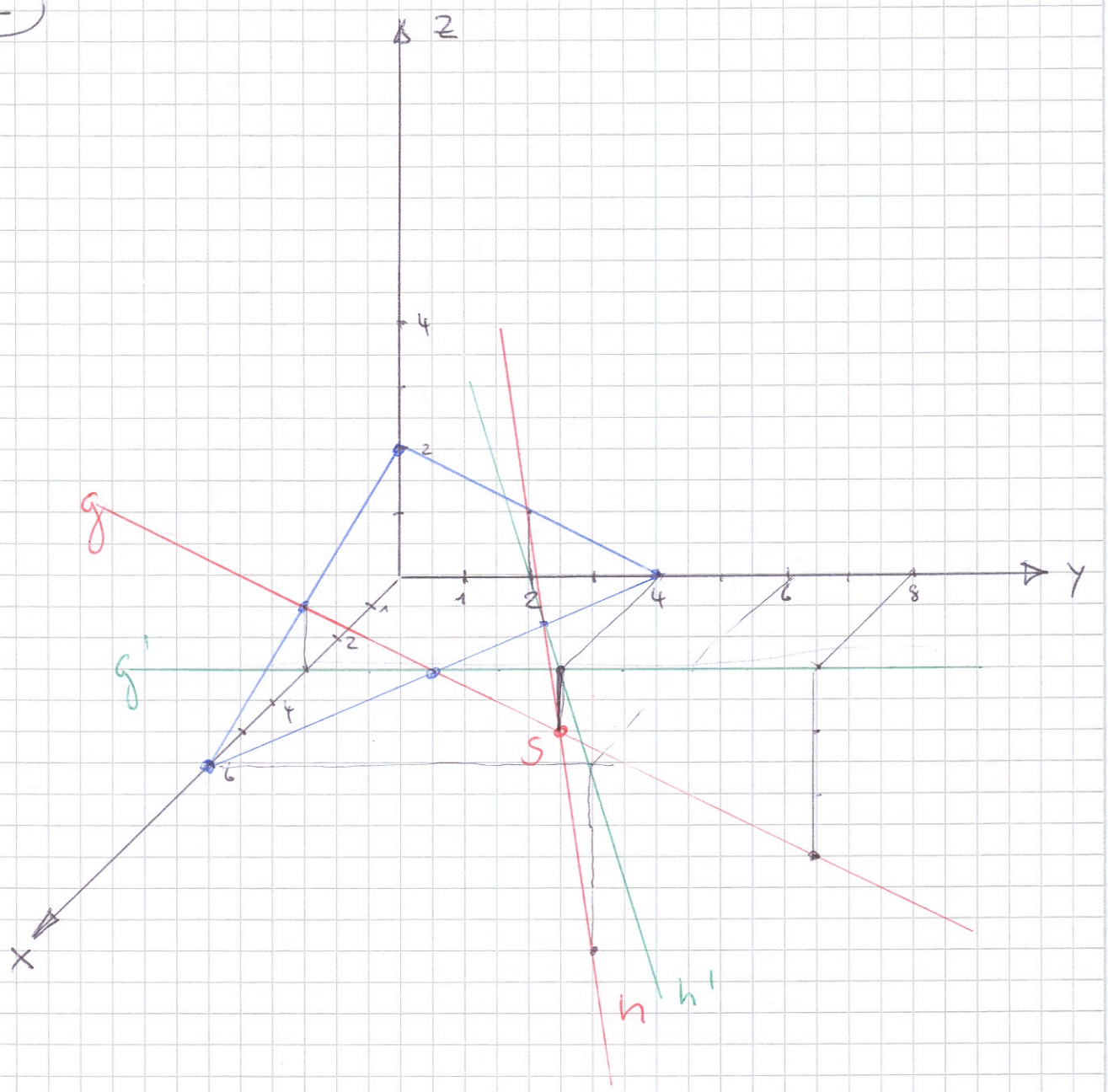
$$\Rightarrow \quad \vec{n}_F = \vec{n}_E \times \vec{AB}$$

$$\longrightarrow \quad \vec{n}_F = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$F: \quad x + 2y + 2z - 4 = 0$$

$$\vec{AB} = \begin{pmatrix} -10 \\ 3 \\ 2 \end{pmatrix}$$

(2)



$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ -3 \end{pmatrix} + s \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \quad ; \quad P_1(3|8|-3)_{s=0} \; ; \; P_2(3|0|1)_{s=-4}$$

$$h: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \qquad P_1(0|2|1)_{t=2} \quad P_2(6|6|-3)_{t=4}$$

c) $g = h$: $S(3|4|-1)$

b) $\vec{v}_g = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$, $\vec{v}_h = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$ (Richtungsvek. von g und h)

$$\cos\varphi = \frac{\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}}{\sqrt{2^2+1^2}\sqrt{3^2+2^2+2}}$$

$$= \frac{4+2}{\sqrt{5}\sqrt{17}} = \frac{6}{\sqrt{85}} = \frac{6\cdot\sqrt{85}}{85}$$

$$\arccos\left(\frac{6\cdot\sqrt{85}}{85}\right) = 49.4° \quad (49.3987054)$$

d) $\vec{v}_g \times \vec{v}_h = \begin{pmatrix} -2 \\ -3 \\ -6 \end{pmatrix} \longrightarrow \vec{n}_E = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$

$E: 2x + 3y + 6z + D = 0$

$P(0|2|1)$ eins. $\rightarrow$ aus a)

$2\cdot 0 + 3\cdot 2 + 6\cdot 1 + D = 0$

$6 + 6 + D = 0$

$D = -12$

$\underline{2x + 3y + 6z - 12 = 0}$

x-Achse: $2x - 12 = 0 \Rightarrow x_0 = 6$

y-Achse: $3y - 12 = 0 \Rightarrow y_0 = 4$

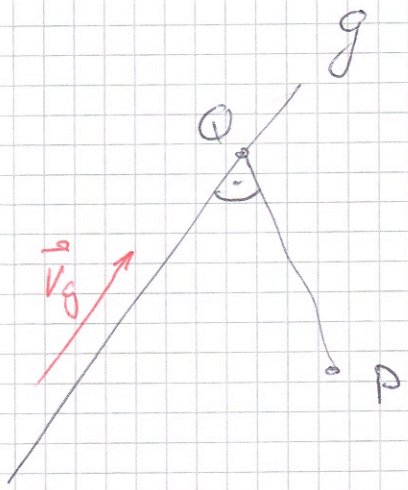
z-Achse: $6z - 12 = 0 \Rightarrow z_0 = 2$

(5) $P(4/5/5)$

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$Q \in g \Rightarrow \vec{r}_Q = \begin{pmatrix} -2s-4 \\ 2s+4 \\ s \end{pmatrix}$$

$$\vec{v}_g = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$



$$\vec{PQ} \cdot \vec{v}_g = 0$$

$$(\vec{r}_Q - \vec{r}_P) \cdot \vec{v}_g = 0$$

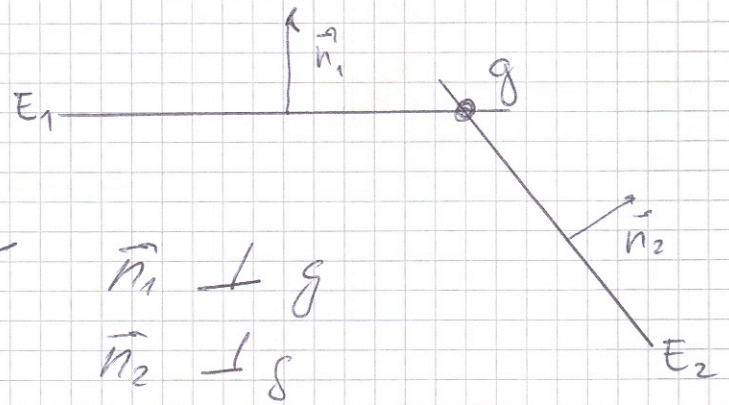
$$\begin{pmatrix} -2s-8 \\ 2s-1 \\ s-5 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} = 0$$

$$9s + 9 = 0$$

$$s = -1 \Rightarrow Q(-2/2/-1)$$

$$|\vec{PQ}| = 9$$

(7)



~~g ⊥~~ $\vec{n}_1 \perp g$

$\vec{n}_2 \perp g$

$\Rightarrow g \parallel \vec{n}_1 \times \vec{n}_2$

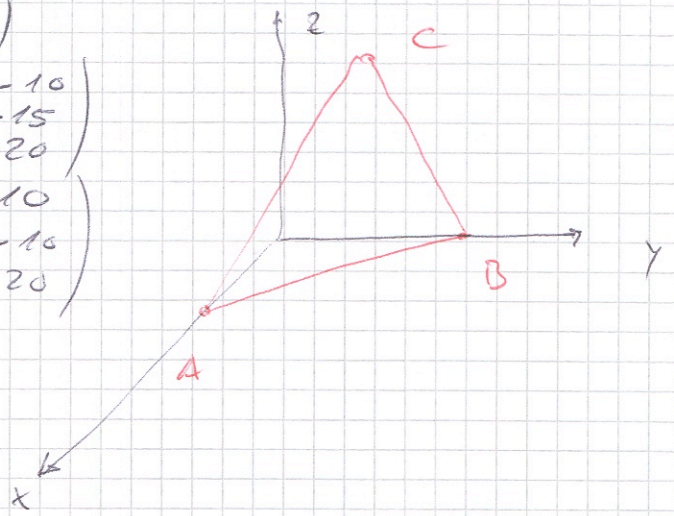
(6) $A(x/0/0)$

$B(0/y/0)$; $y = x+5 \Rightarrow B(0/x+5/0)$

$C(10/15/20)$

$$\vec{CA} = \vec{r_A} - \vec{r_C} = \begin{pmatrix} x-10 \\ -15 \\ -20 \end{pmatrix}$$

$$\vec{CB} = \vec{r_B} - \vec{r_C} = \begin{pmatrix} -10 \\ x-10 \\ -20 \end{pmatrix}$$



$$\frac{1}{2} \left| \begin{pmatrix} x-10 \\ -15 \\ -20 \end{pmatrix} \times \begin{pmatrix} -10 \\ x-10 \\ -20 \end{pmatrix} \right| = \sqrt{7149}$$

$$\frac{1}{2} \left| \begin{pmatrix} 20x+100 \\ 20x \\ x^2-20x-50 \end{pmatrix} \right| = \sqrt{7149}$$

$$\frac{1}{2} \sqrt{(20x+100)^2 + (20x)^2 + (x^2-20x-50)^2} = \sqrt{7149} \quad / \; x^2$$

$$\frac{1}{4} \left((20x+100)^2 + (20x)^2 + (x^2-20x-50)^2 \right) = 7149 \; / \cdot 4$$

$$x^4 - 40x^3 + 1100x^2 + 6000x + 12500 = 4 \cdot 7149$$

$x_1 = -6.2$ $\quad y_1 = -1.2$

$x_2 = 2$ $\quad y_2 = 7$

$A_1(-6.2/0/0), B_1(0/-1.2/0)$

$A_2(2/0/0), B_2(0/7/0)$