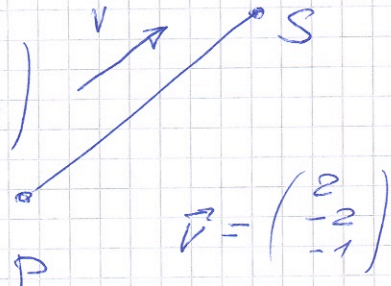
VG 7E, 11.9.13

(1) $\vec{r}_P = \vec{r}_S - 15 \cdot \frac{\vec{v}}{|\vec{v}|}$

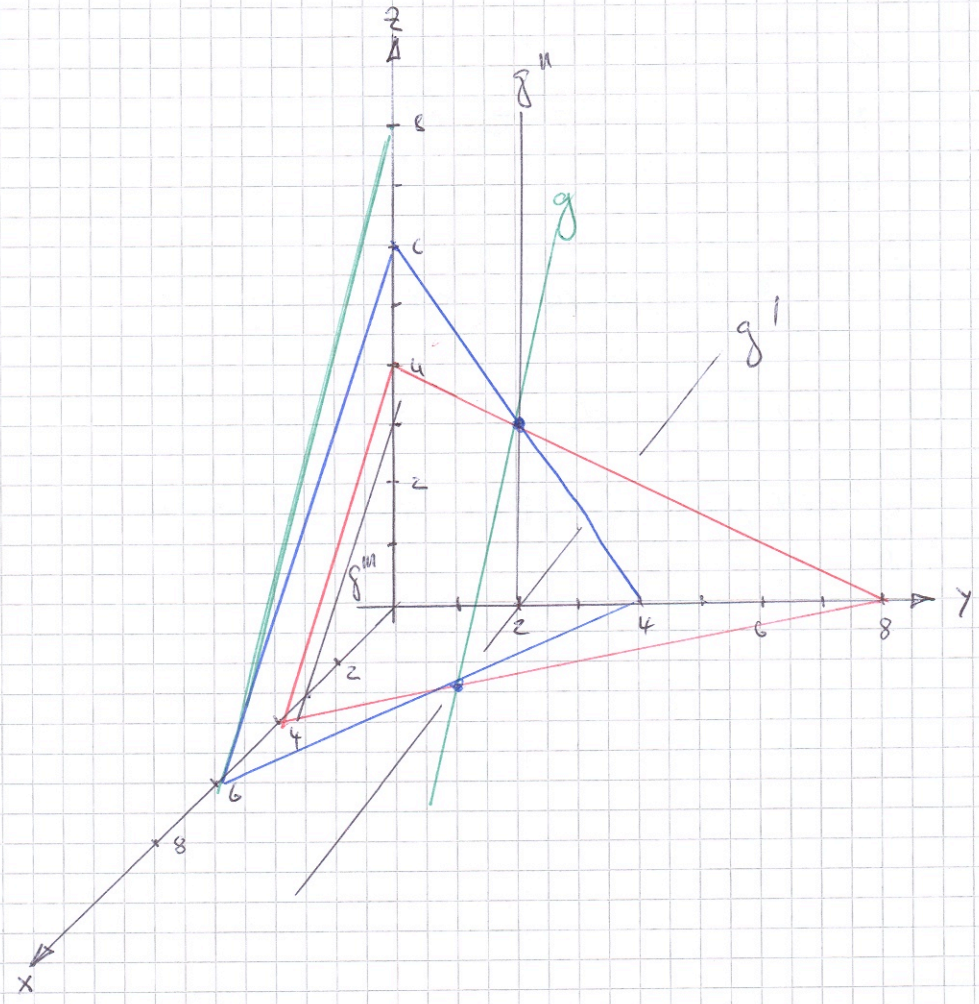
$= \begin{pmatrix} 13 \\ -3 \\ -3 \end{pmatrix} - 15 \cdot \frac{1}{\sqrt{2^2+2^2+1^2}} \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 13 \\ -3 \\ -3 \end{pmatrix} - 5 \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix}$



$P(3/7/2)$

(2)



(2b) $\vec{n_1} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$; $\vec{n_2} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$

$$\varphi = \arccos\left(\frac{\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}}{3 \cdot \sqrt{17}} \right)$$

$$\frac{4+3+4}{3\sqrt{17}} = \frac{11}{3\sqrt{17}}$$

$$\arccos\left(\frac{11}{3 \cdot \sqrt{17}} \right) = \underline{\underline{27.215°}}$$

c) $x = x$

$y = 2$

$z = -x + 3$

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$\vec{g} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ parallel xz-Ebene

$P_0(0|2|3) \notin$ xz-Ebene

g ist parallel zu xz-Ebene

(3) $\vec{r_R} = \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix}$; $\vec{r_S} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$; $\vec{r_P} = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$

$\vec{PR} \perp \vec{PS}$

$\vec{PR} \cdot \vec{PS} = 0$

$(\vec{r_R} - \vec{r_P}) \cdot (\vec{r_S} - \vec{r_P}) = 0$

$\begin{pmatrix} 1 \\ 2 \\ 8-z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 3-z \end{pmatrix} = 0$

$2 + 4 + (8-z)(3-z) = 0$

$6 + 24 - 11z + z^2 = 0$

$z^2 - 11z + 30 = 0$

$(z-5)(z-6) = 0$

$z_1 = 5 \quad / \quad z_2 = 6$

(4) $\vec{v} = \begin{pmatrix} 8 \\ 2 \\ 1 \end{pmatrix}$; $\vec{w} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

E_1 durch $P_1(-1/1/-2)$:

$\vec{v} \times \vec{w} = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix}$; $E_1: x - 5y + 2z + D = 0$

P_1 eins.: $-1 - 5 \cdot 1 + 2(-2) + D = 0$

$-1 - 5 - 4 + D = 0$

$D = 10$

$E_1: x - 5y + 2z + 10 = 0$

HNF: $\pm d = \dfrac{x - 5y + 2z + 10}{\sqrt{30}}$

$P_2(12/4/9)$ eins.: $\dfrac{12 - 5 \cdot 4 + 2 \cdot 9 + 10}{\sqrt{30}} =$

$= \dfrac{20}{\sqrt{30}} = \dfrac{20 \cdot \sqrt{30}}{30}$

$= \underline{\underline{\dfrac{2}{3}\sqrt{30} \approx 3.65}}$

(5) $E: 3x - y + 2z - 5 = 0$

$\vec{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

HNF: $\pm d = \frac{3x - y + 2z - 5}{\sqrt{14}}$

$Q(13/-2/3)$ eins.:

$$\frac{3 \cdot 13 - (-2) + 2 \cdot 3 - 5}{\sqrt{14}}$$

$$= \frac{39 + 2 + 6 - 5}{\sqrt{14}} = \frac{42}{\sqrt{14}} = 3\sqrt{14} > 0$$

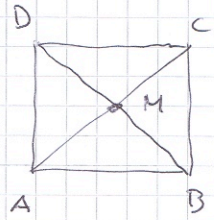
$$\vec{r}_R = \vec{r}_Q - 4 \cdot 3 \cdot \sqrt{14} \, \frac{\vec{n}}{n}$$

$$= \begin{pmatrix} 13 \\ -2 \\ 3 \end{pmatrix} - 12 \cdot \sqrt{14} \cdot \frac{1}{\sqrt{14}} \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 13 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 36 \\ -12 \\ 24 \end{pmatrix} = \begin{pmatrix} -23 \\ 10 \\ -21 \end{pmatrix} \Rightarrow \underline{\underline{R(-23/10/-21)}}$$

(6) $B(5|0|10)$, $D(2|21|10)$

$A(x|y|0)$



Lösung mittels Längen:

$$\overline{AB} = \overline{AD}$$

$$|\vec{AB}| = |\vec{AD}|$$

$$\left|\begin{pmatrix} 5-x \\ -y \\ 10 \end{pmatrix}\right| = \left|\begin{pmatrix} 2-x \\ 21-y \\ 10 \end{pmatrix}\right|$$

$$x^2 - 10x + y^2 + 125 = x^2 - 4x + y^2 - 42y + 545$$

$$\underline{\underline{6x - 42y + 420 = 0}}$$

$$\vec{r}_M = \frac{1}{2}\left(\vec{r}_B + \vec{r}_D\right) = \begin{pmatrix} 7/2 \\ 21/2 \\ 10 \end{pmatrix}$$

$$\overline{MA} = \overline{MB} \Rightarrow |\vec{MA}| = |\vec{MB}|$$

$$\left|\begin{pmatrix} x - \frac{7}{2} \\ y - \frac{21}{2} \\ -10 \end{pmatrix}\right| = \left|\begin{pmatrix} \frac{3}{2} \\ -\frac{21}{2} \\ 0 \end{pmatrix}\right|$$

$$x^2 - 7y + y^2 - 21y + \frac{445}{2} = \frac{225}{2}$$

$$\underline{\underline{x^2 - 7x + y^2 - 21y + 110 = 0}}$$

$x_1 = 0,\ y_1 = 10$ $\quad A_1(0|10|0)$

$x_2 = 7,\ y_2 = 11$ $\quad A_2(7|11|0)$

(6) Lösung mit Winkeln:

$\vec{AB} \cdot \vec{AD} = 0$ (rechter Winkel)

$$\begin{pmatrix} 5-x \\ -7 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 2-x \\ 21-7 \\ 10 \end{pmatrix} = 0$$

$$x^2 - 7x + y^2 - 21y + 110 = 0$$

$\vec{MA} \cdot \vec{MB} = 0$

$$\begin{pmatrix} x - 7/2 \\ y - 21/2 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} 3/2 \\ -21/2 \\ 0 \end{pmatrix} = 0$$

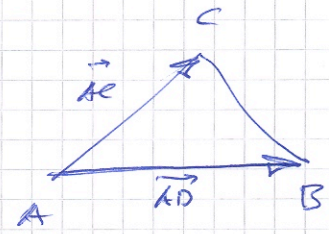
$$3x - 21y + 210 = 0$$

Solve

$A_1(0 \mid 10 \mid 0)$

$A_2(7 \mid 11 \mid 0)$

7



A (2/3/1)

B (-1/2/2)

$A = 2\sqrt{10}$

$\vec{r_C} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$

$A = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right| = 2\sqrt{10}$

$= \frac{1}{2} \left| \begin{pmatrix} -3 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} x-2 \\ -3 \\ -1 \end{pmatrix} \right| = \frac{1}{2} \left| \begin{pmatrix} 4 \\ x-5 \\ x+7 \end{pmatrix} \right|$

$= \frac{1}{2} \sqrt{4^2 + (x-5)^2 + (x+7)^2} = 2 \cdot \sqrt{10}$

$\frac{1}{2} \sqrt{2x^2 + 4x + 90} = 2 \cdot \sqrt{10}$

$\sqrt{2x^2 + 4x + 90} = 4\sqrt{10} \quad / \,^2$

$2x^2 + 4x + 90 = 160$

$2x^2 + 4x - 70 = 0$

$x^2 + 2x - 35 = 0$

$(x-5)(x+7) = 0$

$x_1 = 5 \quad / \quad x_2 = -7$