ML Aufgabenblatt Gravitation, F_Z, Impuls

(1) $F_G = G \frac{m_1 m_2}{R^2} = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2} \frac{1kg \cdot 1kg}{1m^2}$

$= 6.67 \cdot 10^{-11} kg \frac{m}{s^2}$

$= \underline{\underline{6.67 \cdot 10^{-11} N}}$ $\quad 1N = 1 \frac{kg \cdot m}{s^2}$

(2) $m_M = 6.419 \cdot 10^{23} kg$

$R = 3'400'000 m$

$F_G = G \frac{m_M \cdot m}{R^2}$

$\text{Beschleunigung} = \frac{\text{Kraft}}{\text{Masse}} = \frac{F_G}{m}$

$\Rightarrow g_{Mars} = \frac{F_G}{m} = G \frac{m_{Mars}}{R^2} \approx \underline{\underline{3.7 \, m/s^2}}$

(3) R_E = Erdradius, R = ges. Radius

p = Prozentsatz; $p_1 = 0.01$ / $p_2 = 0.1$

$G \frac{M_E}{R_E^2} - G \frac{M_E}{R_E^2} \cdot p = G \frac{M_E}{R^2}$

$G \frac{M_E}{R_E^2}(1-p) = G \frac{M_E}{R^2} \quad \Big| : G : M_E$

$\frac{1-p}{R_E^2} = \frac{1}{R^2}$

$\frac{R_E^2}{1-p} = R^2; \quad R = \frac{R_E}{\sqrt{1-p}}$

~~$p_1 = 0.01$: R~~

~~$p_1 = 0.01$~~:

$$R = \frac{R_E}{\sqrt{1-p}}$$

$$R - R_E = \frac{R_E}{\sqrt{1-p}} - R_E = R_E\left((1-p)^{-1/2} - 1\right)$$

$p_1 = 0.01$: Höhe $\cong$ $\underline{\underline{32.1\,km}}$

$p_2 = 0.1$: Höhe $=$ $\underline{\underline{344.6\,km}}$

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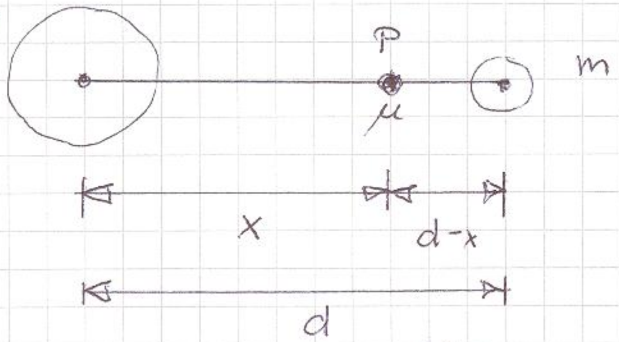


Kräftegleichgewicht in P: $G\,\frac{M\mu}{x^2} = G\,\frac{m\mu}{(d-x)^2}$

$$\frac{M}{x^2} = \frac{m}{(d-x)^2} \quad \Big/ \cdot x^2(d-x)^2$$

$$M(d-x)^2 = mx^2$$

$$Md^2 - 2Mdx + Mx^2 = mx^2$$

$$(M-m)x^2 - 2Mdx + Md^2 = 0$$

$$x_{1,2} = \frac{2Md \pm \sqrt{(2Md)^2 - 4(M-m)Md^2}}{2(M-m)}$$

$$x_{1,2} = \frac{Md \pm d\sqrt{mM}}{M-m} = d\left(\frac{M \pm \sqrt{mM}}{M-m}\right)$$

nur die kleinere Lösung ist relevant:

$$x = d\left(\frac{M - \sqrt{mM}}{M-m}\right)$$

$$x \approx 3.46 \cdot 10^8\,m$$

$$x : d-x \cong 9 : 1$$

(6) Die Energie ist die gleiche wie die, die es braucht, um eine Rakete von der Erdoberfläche in die Unendlichkeit zu schicken. Diese Energie wird mit der kinetischen Energie gleich gesetzt ($\frac{1}{2}mv^2$)

M: Erdmasse m: Satellitenmasse

R: Erdradius

Gravitationskraft: $F_G = G\frac{M \cdot m}{r^2}$

Energie $= F \cdot s$; $F \neq const \Rightarrow E = \int F ds$

$\hookrightarrow F = F_G$

$$E = \int_R^\infty G\frac{Mm}{r^2}dr = GMm\int_R^\infty \frac{1}{r^2}dr$$

$$\left(\int \frac{1}{x^2}dx = \int \frac{dx}{x^2} = = -\frac{1}{x}\right)$$

$$E = GMm\int_R^\infty \frac{1}{r^2}dr = GMm\left[-\frac{1}{r}\right]_R^\infty$$

$$= GMm\left(-\frac{1}{\infty} - \left(-\frac{1}{R}\right)\right) = \frac{GMm}{R} = E_{pot}$$

$$E_{pot} = E_{kin}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} \quad \Big| : m \cdot 2$$

$$v = \sqrt{\frac{2GM}{R}}$$

(7) $F_Z = \frac{mv^2}{R} = m\omega^2 R$

Je nach Bedarf die geeignete Formel verwenden:

a) $v' = 2v$ $\quad F_Z' = \frac{mv'^2}{R'} = \frac{m(2v)^2}{R}$

$R' = R$

$$= 4 \cdot \frac{mv^2}{R} = 4 \cdot F_Z$$

$$\underline{\underline{F_Z' = 4 \cdot F_Z}}$$

b) $v' = v$, $R' = 2R$

$F_Z = \frac{mv^2}{R}$; $F_Z' = \frac{mv'^2}{R'} = \frac{mv^2}{2R} = \frac{1}{2} \cdot \frac{mv^2}{R}$

$$\underline{\underline{F_Z' = \frac{1}{2} F_Z}}$$

c) $R' = R$; $T' = 2T$;

$\omega = \frac{2\pi}{T}$; $F_Z = m\omega^2 R = m\left(\frac{2\pi}{T}\right)^2 R$

$$F_Z = (2\pi)^2 \frac{mR}{T^2}$$

$F_Z' = m\omega'^2 R' = m\left(\frac{2\pi}{T'}\right)^2 R' = m\left(\frac{2\pi}{2T}\right)^2 R'$

$F_Z' = \pi^2 \frac{mR}{T^2}$

$\Rightarrow$ $\underline{\underline{F_Z' = \frac{1}{4} F_Z}}$

d) $F_Z = m\omega^2 R$; $\omega' = 2\omega \Rightarrow F_Z' = 4F_Z$

e) Tourenzal prop. $\omega \Rightarrow$ wie d)

(8) $R_E = 6.371 \cdot 10^6 m$

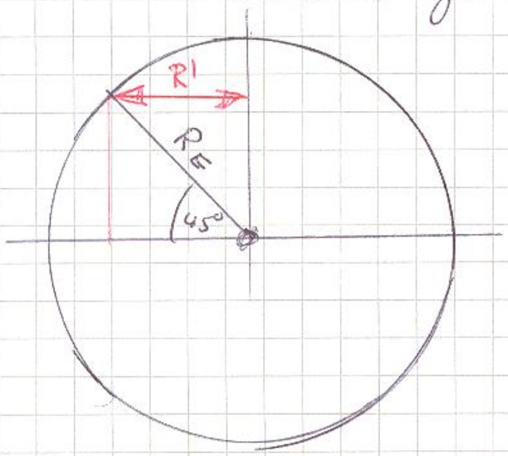
$F_Z = m\omega^2 R \; ; \; a_Z = \frac{F_Z}{m} = \omega^2 R$

$a_Z = \omega^2 R = \left(\frac{2\pi}{24h}\right)^2 R_E$ $\quad T = 24h, \omega = \frac{2\pi}{T}$

$= \frac{(2\pi)^2}{(24 \cdot 60^2)^2} R_E \cong 0.0337 \, m/s^2$

a_Z wirkt der Erdbeschleunigung $g = 9.81 \, m/s^2$ entgegen!

45. Breitengrad:



$\cos 45° = \frac{R'}{R_E} \Leftrightarrow R' = R_E \cos 45°$

$a_Z' = \omega^2 R' = \left(\frac{2\pi}{24 \cdot 60^2}\right)^2 R_E \cdot \cos 45°$

$a_Z' = 0.0238 \, m/s^2$

(9) Radius $R = R_E + 2'000'000 m = 8.371 \cdot 10^6 m$

$F_Z = F_G$

$m\omega^2 R = G \frac{mM}{R^2} \quad / : m : R$

$\omega^2 = G \frac{M}{R^3}$

$\omega = \sqrt{\frac{GM}{R^3}} \; ; \; \omega = \frac{2\pi}{T} \; ; \; T = \frac{2\pi}{\omega}$

$T = 2\pi \sqrt{\frac{R^3}{GM}} \cong$ <u>2.12 Stunden</u>

$v = \omega R \cong 6'902.7 \, m/s \cong 24'849.7 \, km/h$

(10) $v = 3'667\ km/h$

$$F_Z = \frac{mv^2}{R} = \cancel{m}\ G\,\frac{mM}{R^2} \quad \Big/ \cdot R^2 \ : m$$

$$v^2 \cdot R = GM$$

$$R = \frac{GM}{v^2} = \cancel{3.78 \cdot 10^8}\ m$$

$$3.84 \cdot 10^8 m - R_E$$

$$= 3.78 \cdot 10^8 m$$

(Erdradius abziehen für Höhe über Erdoberfläche)

(11) $F_Z = F_G \quad M = M_{Sonne}$

$$m\omega^2 R = G\,\frac{mM}{R^2} \quad \Big/ : m \ \cdot R^2$$

$$\omega^2 R^3 = GM$$

$$R^3 = \frac{GM}{\omega^2}$$

$$R = \sqrt[3]{\frac{GM}{\omega^2}} = \sqrt[3]{\frac{GM}{\left(\frac{2\pi}{365.25 \cdot 24 \cdot 60^2}\right)^2}}$$

$$= 1.496 \cdot 10^{11} m$$

= Abstand Erde – Sonne

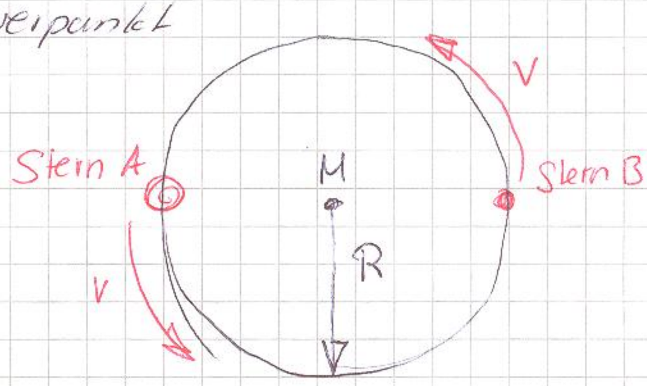
(12) $d = 10\,Lj.$ $M := $ Schwerpunkt

Zu beachten:

Abstand der Sterne: $d = 2R$

aber:

Kreisradius $= R$!



Gleichgewicht: $F_{Grav.} = F_{Zentripetal}$

$$G\,\frac{m_1 m_2}{d^2} = m_1 \omega^2 R$$

$$m_1 = m_2 = m$$

$$G\,\frac{m^2}{(2R)^2} = m\omega^2 R \quad \Big| : m$$

$$\frac{Gm}{4R^2} = \omega^2 R \quad \Big| : R$$

$$\frac{Gm}{4R^3} = \omega^2 = \left(\frac{2\pi}{T}\right)^2 \quad \text{gesucht ist } T$$

$$\frac{Gm}{4R^3} = \frac{(2\pi)^2}{T^2} \quad \Big| \; 1/x$$

$$\frac{4R^3}{Gm} = \frac{T^2}{(2\pi)^2} \quad \Big| \cdot (2\pi)^2$$

$$\frac{(2\pi)^2 4R^3}{Gm} = T^2$$

$$\sqrt{\frac{16\pi^2 R^3}{Gm}} = T$$

$$4\pi\sqrt{\frac{R^3}{Gm}} = T \mathrel{\hat{=}} 31.83 \text{ Mio. Jahre}$$

$$1\,Lj. = 3\cdot 10^8\,\frac{m}{s} \cdot 365 \cdot 24 \cdot 60^2\,s$$

(13) Satellit 1 : m_1 , w_1 , T_1 , R_1 | $M = M_{Erde}$

Satellit 2 : m_2 , w_2 , T_2 , R_2

Es gilt für beide : $F_Z = F_G$

$$m_1 \omega_1^2 R_1 = \frac{G m_1 M}{R_1^2} \qquad m_2 \omega_2^2 R_2 = \frac{G m_2 M}{R_2^2}$$

$$\omega_1^2 R_1 = \frac{GM}{R_1^2} \quad \Big| : R_1 \qquad \omega_2^2 R_2 = \frac{GM}{R_2^2} \quad \Big| : R_2$$

$$\omega_1^2 = \frac{GM}{R_1^3} \qquad \omega_2^2 = \frac{GM}{R_2^3}$$

$$\frac{(2\pi)^2}{T_1^2} = \frac{GM}{R_1^3} \qquad \frac{(2\pi)^2}{T_2^2} = \frac{GM}{R_2^3}$$

Gleichung $\underline{I}$: Gleichung $\underline{II}$

$$\frac{\frac{\cancel{(2\pi)^2}}{T_1^2}}{\frac{\cancel{(2\pi)^2}}{T_2^2}} = \frac{\frac{\cancel{GM}}{R_1^3}}{\frac{\cancel{GM}}{R_2^3}}$$

$$\frac{T_2^2}{T_1^2} = \frac{R_2^3}{R_1^3} \quad \Big| \ 1/x$$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \iff \boxed{T_1^2 : T_2^2 = R_1^3 : R_2^3}$$

Prosaisch:

Die Quadrate der Umlaufzeiten zweier Satelliten (oder Planeten) verhalten sich wie die dritten Potenzen der Bahnradien

$R_1 = 35'879'085\,m + 6'371'000\,m = 42'250'085\,m$ | 24h

$R_2 = 93'629'000\,m +$ " $= 100'000'000\,m$ | x h

$$T_1^2 : T_2^2 = R_1^3 : R_2^3 \iff \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \iff \frac{T_2^2}{T_1^2} = \frac{R_2^3}{R_1^3}$$

$$\iff T_2 = T_1 \sqrt{(R_2/R_1)^3} \cong \underline{\underline{87.4\,h}}$$

(13) Fortsetzung

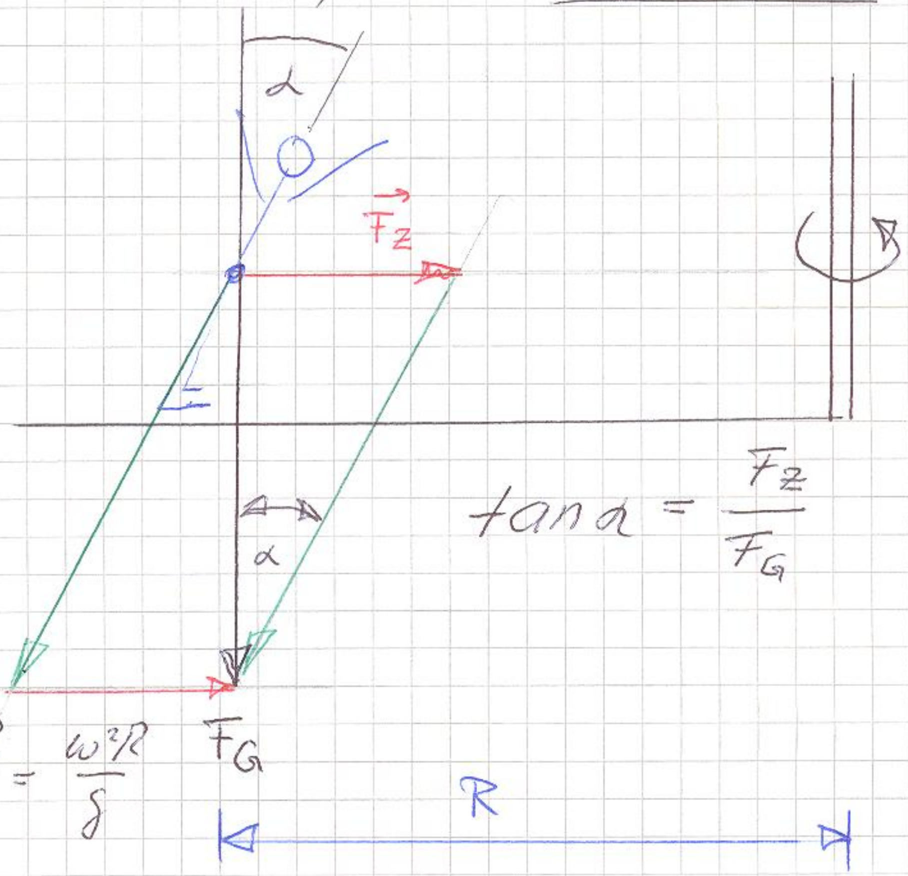
$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \quad \Big/ \ 1/x$$

$$\frac{T_2^2}{T_1^2} = \frac{R_2^3}{R_1^3} \quad \Big/ \cdot T_1^2$$

$$T_2 = \sqrt{T_1^2 \frac{R_2^3}{R_1^3}} = T_1 \cdot \sqrt{\left(\frac{R_2}{R_1}\right)^3}$$

$$T_2 = T_1 \left(R_2/R_1\right)^{3/2} \cong \underline{\underline{87.4 \text{ Stunden}}}$$

(14)



$$\tan\alpha = \frac{F_Z}{F_G}$$

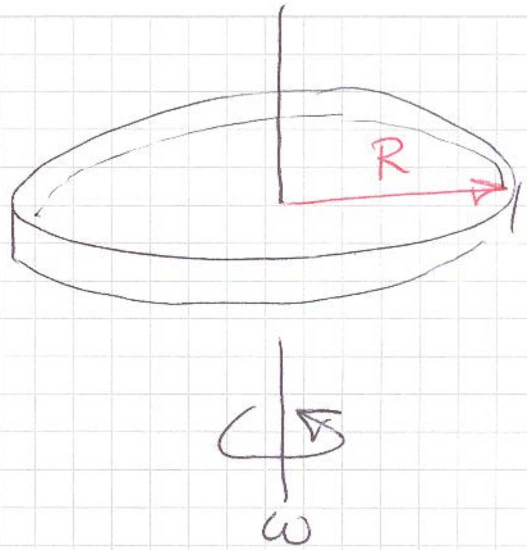
$$\tan\alpha = \frac{m\omega^2 R}{mg} = \frac{\omega^2 R}{g}$$

$$\frac{g \cdot \tan\alpha}{R} = \omega^2 = \left(\frac{2\pi}{T}\right)^2 \quad \Big/ \ 1/x$$

$$\frac{R}{g \cdot \tan\alpha} = \frac{1}{\omega^2} = \frac{T^2}{(2\pi)^2} \quad \Big/ \cdot (2\pi)^2$$

$$\sqrt{\frac{(2\pi)^2 \cdot R}{g \cdot \tan\alpha}} = 2\pi \sqrt{\frac{R}{g \tan\alpha}} \cong \underline{\underline{8.8 s}}$$

(15) Moonraker:



$$\omega^2 R = g$$

$$\omega^2 = \frac{g}{R}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{R} \quad \Big| \ 1/x$$

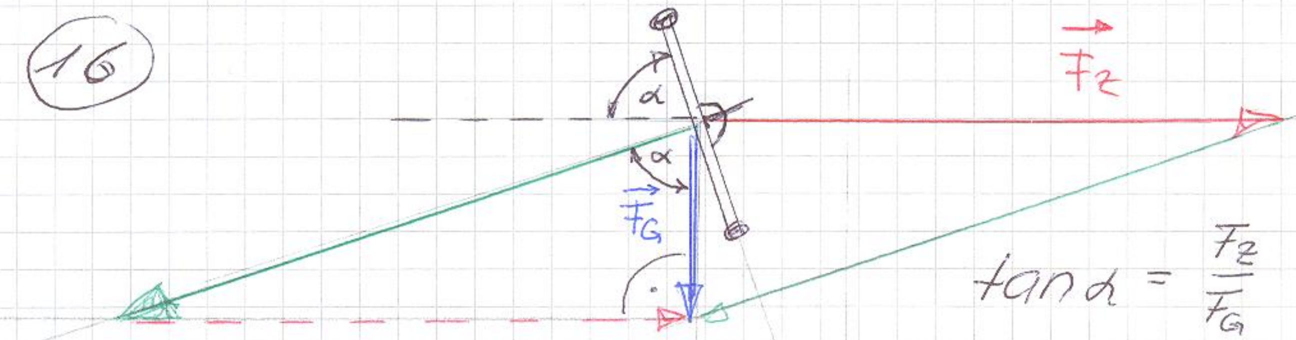
$$\frac{T^2}{(2\pi)^2} = \frac{R}{g}$$

$$T = 2\pi\sqrt{R/g} \cong \underline{\underline{24.57s}}$$

$$f = \frac{1}{T} = \underline{\underline{0.041\,Hz}}$$

$$v = \omega \cdot R = \frac{2\pi}{T} \cdot R \cong 38.36\,m/s \cong 138.1\,km/h$$

(16)



$$\tan\alpha = \frac{F_Z}{F_G}$$

Kurvenradius: $F_Z = \frac{mv^2}{R} = 10mg \Rightarrow R = \frac{v^2}{10g} \cong \underline{\underline{4530.5m}}$

Winkel: $\tan\alpha = \frac{F_Z}{F_G} = \frac{\frac{mv^2}{R}}{mg} = \frac{v^2}{gR}$

$R = \frac{v^2}{10g}$ einsetzen: $\tan\alpha = \frac{v^2}{g \cdot \frac{v^2}{10g}} = 10$

$$\tan\alpha = 10$$

$$\underline{\underline{\alpha = 84.29°}}$$

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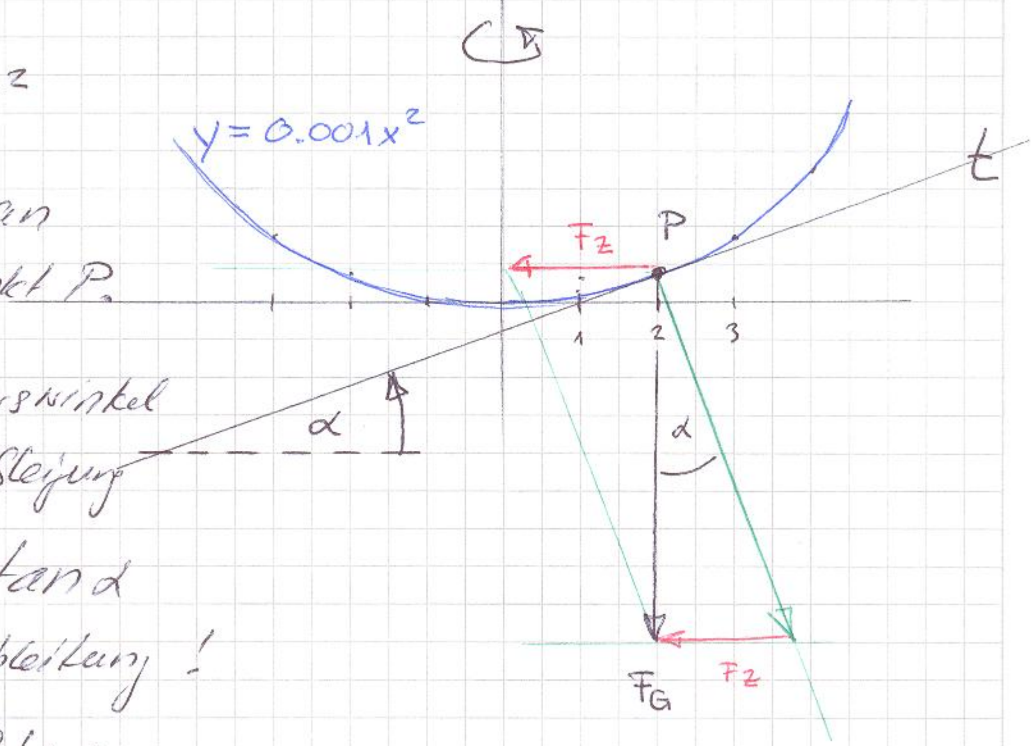
$f(x) = 0.001x^2$



t ist Tangente an f(x) im ~~Pu~~ Punkt P.

α ist der Steigungswinkel

tan α ist die Steigung

↪ $m = \tan\alpha$

Steigung = Ableitung!

$m(x) = f'(x)$:

Steigung bei $x = 2$: $f'(x) = 0.002x$

$$\underline{f'(2) = 0.004 = \tan\alpha}$$

$$\frac{F_Z}{F_G} = \tan\alpha = 0.004$$

$$\frac{F_Z}{F_G} = \frac{m\omega^2 R}{mg} = \frac{2\omega^2}{g} \longrightarrow R = 2 \text{ im Punkt } P(2/f(2))$$

$$\Longrightarrow \frac{F_Z}{F_G} = \frac{2\omega^2}{g} = 0.004$$

$$\omega^2 = \frac{0.004g}{2} = 0.002g$$

$$\left(\frac{2\pi}{T}\right)^2 = 0.002g$$

$$\frac{(2\pi)^2}{T^2} = 0.002g$$

$$T = 2\pi\sqrt{\frac{1}{0.002g}} = \underline{\underline{44.86\,s}}$$

(18) $m_1 = 80kg \ , \ v_1 = 3 \, m/s$

$m_2 = x \ , \ v_2 = -8 \, m/s$

$P_{initial} = \cancel{m_1 v_1} \ 0$

$P_{final} = m_1 v_1 + m_2 v_2$

$$p_i = p_f$$

$$m_1 v_1 + m_2 v_2 = 0$$

$$\cancel{m_1 \quad = - m_2}$$

$$m_2 = -m_1 \frac{v_1}{v_2} = -80kg \cdot \frac{3}{-8} = \underline{\underline{30kg}}$$

(19) $p = 0.01kg \cdot 400 \, m/s = 70kg \cdot x$

$$\frac{0.01}{70} \cdot 400 \frac{m}{s} = 0.057 \frac{m}{s}$$

Der Geschossene bewegt sich mit $0.057 \frac{m}{s}$

↳ sehr kleiner Impuls

einfacher: actio = reactio

das Opfer erhält gleiche Kraft wie Schütze durch Rückstoss, und der Schütze fällt ja auch nicht um

(20) Hinweis: Die folgende Berechnung ist physikalisch nicht ganz korrekt. Die Aufgabe dient der Veranschaulichung des Raketenantriebs.

$M = 70\,kg$, $m_{1,2,3} = 5\,kg$, $v_{Stein} = 8\,m/s$

1. Stein: $m_{Skater} = 70\,kg + 2 \cdot 5\,kg = 80\,kg$

wirft Stein Nr. 1 weg: $p_{Stein} = 5\,kg \cdot 8\,\frac{m}{s} = 40\,kg\,m/s$

Impuls Stein = Impulsänderung Skater

$$80\,kg \cdot \Delta v = 40\,kg\,m/s$$

$$\Rightarrow \underline{\underline{\Delta v = 0.5\,m/s}}$$

2. Stein: $m_{Skater} = 70\,kg + 5\,kg = 75\,kg$

$$75\,kg \cdot \Delta v = 5\,kg \cdot 8\,m/s = 40\,kg\,m/s$$

$$\Delta v = \frac{40}{75}\,m/s = \underline{\underline{0.5\overline{3}\,m/s}}$$

3. Stein: $m_{Skater} = 70\,kg$

$$70\,kg \cdot \Delta v = 40\,kg\,m/s$$

$$\Delta v = \frac{40}{70}\,m/s = \frac{4}{7}\,m/s \cong \underline{\underline{0.57\,m/s}}$$

$$\Delta v_{tot} = 0.5\,m/s + 0.5\overline{3}\,m/s + 0.57\,m/s \cong \underline{\underline{1.6\,m/s}}$$

b) $\Delta p = 3 \cdot 5\,kg \cdot 8\,\frac{m}{s} = 70\,kg \cdot \Delta v$

$$120\,kg\,m/s = 70\,kg \cdot \Delta v$$

$$\underline{\underline{1.71\,m/s \cong \Delta v}}$$

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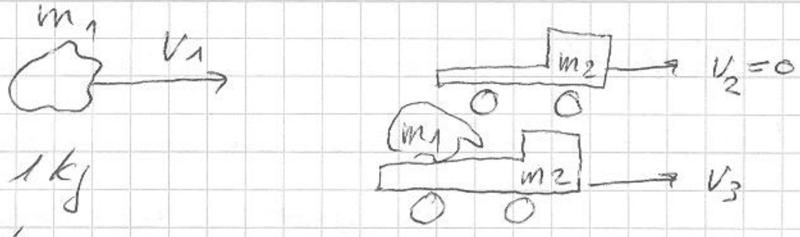


$m_1 = 0.1\,kg$

$m_2 = 1\,kg$

$V_1 = 8\,m/s$

Impulserhaltung:

$$m_1 V_1 = (m_1 + m_2) V_3$$

$$\frac{m_1 V_1}{m_1 + m_2} = V_1 \cdot \frac{m_1}{m_1+m_2} = V_3 = 0.091\,m/s \cdot 8$$

$$= 0.\overline{72}\,m/s$$

Energie vorher: $E_i = \frac{1}{2} m_1 V_1^2 = 3.2\,J$.

Energie nachher: $E_f = \frac{1}{2}(m_1 + m_2) V_3^2 = 0.291\,J$

$\hookrightarrow$ 9.1 % der Bewegungs-Energie erhalten!

c) $E_i = \frac{1}{2} m_1 v_1^2$ $\quad E_f = \frac{1}{2}(m_1 + m_2) V_3^2$

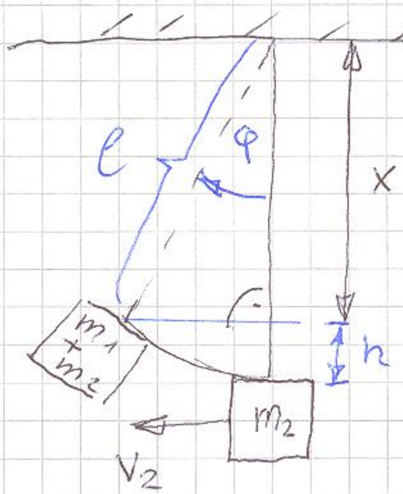
$$= \frac{1}{2}(m_1 + m_2)\left(V_1 \frac{m_1}{m_1+m_2}\right)^2$$

$$= \frac{1}{2} \frac{m_1^2}{m_1+m_2} V_1^2$$

$$\frac{E_f}{E_i} = \frac{\frac{1}{2} \frac{m_1^2}{m_1+m_2} V_1^2}{\frac{1}{2} m_1 V_1^2} = \frac{m_1}{m_1+m_2}$$

$$\text{Prozentsatz} = \underline{\underline{100 \cdot \frac{m_1}{m_1+m_2}}}$$

22 Ballistisches Pendel

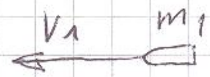


$$\cos\varphi = \frac{x}{l}$$

$$x = l \cdot \cos\varphi$$

$$h = l - l\cos\varphi$$

$$h = l(1-\cos\varphi)$$



$$p_i = m_1 v_1$$

$$p_f = (m_1+m_2) v_2$$

$$v_2 = v_1 \cdot \frac{m_1}{m_1+m_2}$$

Nach dem Schuss in den Block wird die kinetische Energie von Kugel + Block in potentielle Energie (mgh) gewandelt. Hier gilt wieder Energieerhaltung:

$$E_{kin} = E_{pot}$$

$$\frac{1}{2}(m_1+m_2)\left(v_1 \frac{m_1}{m_1+m_2}\right)^2 = (m_1+m_2) \cdot g \cdot h \quad \Big| :(m_1+m_2)$$

$$\frac{1}{2} v_1^2 \left(\frac{m_1}{m_1+m_2}\right)^2 = gh \quad \Big| \cdot 2$$

$$v_1^2 \frac{m_1^2}{(m_1+m_2)^2} = 2gh \quad \Big| \cdot \frac{(m_1+m_2)^2}{m_1^2}$$

$$v_1^2 = 2gh \cdot \frac{(m_1+m_2)^2}{m_1^2} \quad \Big| \sqrt{}$$

$$v_1 = \sqrt{2gh \cdot \left(\frac{m_1+m_2}{m_1}\right)^2}$$

$$= \frac{m_1+m_2}{m_1} \cdot \sqrt{2gh}$$

$$\boxed{v_1 = \frac{m_1+m_2}{m_1}\sqrt{2gl(1-\cos\varphi)}}$$

(23) $m_i = 2'000\,kg$, $m_f = 1'000\,kg$

$v_G = 4'000\,m/s$, $\Delta v = 1000\,m/s$

$$\Delta v = v_G \cdot \ln\left(\frac{m_i}{m_f}\right)$$

$$\frac{\Delta v}{v_G} = \ln(m_i/m_f) \quad \Big/ e^x$$

$$e^{\frac{\Delta v}{v_G}} = \frac{m_i}{m_f} \quad \Big/ \cdot e^{-\frac{\Delta v}{v_G}} \cdot m_f$$

$$m_f = m_i e^{-\frac{\Delta v}{v_G}}$$

$$\text{Treibstoffmenge} = m_i - m_f$$

$$= m_i - m_i e^{-\Delta v / v_G}$$

$$m_{Treibstoff} = m_i\left(1 - e^{-\frac{\Delta v}{v_G}}\right) \hat{=} \underline{\underline{442.4\,kg}}$$

(24) $m_i = 800\,kg$, $v_G = 2'039.1\,m/s$, $\Delta v = 2'600\,m/s$

aus (23): $m_f = m_i e^{-\frac{\Delta v}{v_G}} \hat{=} 300\,kg$

(25) 80% Treibstoff, $v_G = 4'000\,m/s$, $\Delta v = ?$

$m_i = 100\%$, $m_f = 20\%$

$$\Rightarrow m_i/m_f = 5$$

$$\Delta v = v_G \cdot \ln\left(\frac{m_i}{m_f}\right) = v_G \cdot \ln 5$$

$$\hat{=} \underline{\underline{6'437.75\,m/s}}$$