ML Kreisbewegungen G3d/e

(1)

a) $v' = 2v$

$F_Z = \frac{mv^2}{R}$, $F_Z' = \frac{mv'^2}{R} = \frac{m(2v)^2}{R}$

$= \frac{4mv^2}{R} = 4 \cdot \frac{mv^2}{R}$

$\Rightarrow \underline{\underline{F_Z' = 4F_Z}}$

b) $R' = 2R$;

$F_Z = \frac{mv^2}{R}$

$F_Z' = \frac{mv^2}{R'} = \frac{mv^2}{2R} = \frac{1}{2} \cdot \frac{mv^2}{R} = \frac{1}{2} F_Z$

$\underline{\underline{F_Z' = \frac{1}{2} F_Z}}$

c) $T' = 2T$, $\omega = \frac{2\pi}{T}$

$F_Z = m\omega^2 R = m\left(\frac{2\pi}{T}\right)^2 R = m \frac{(2\pi)^2}{T^2} R$

$F_Z' = m\left(\frac{2\pi}{T'}\right)^2 R = m\left(\frac{2\pi}{2T}\right)^2 R = m \frac{(2\pi)^2}{4T^2} R$

$= m \frac{1}{4} \frac{(2\pi)^2}{T^2} R = \frac{1}{4} m \frac{(2\pi)^2}{T^2} R = \frac{1}{4} F_Z$

$\underline{\underline{F_Z' = \frac{1}{4} F_Z}}$

d) $\omega' = 2\omega$

$F_Z = m\omega^2 R$

$F_Z' = m\omega'^2 R = m(2\omega)^2 R = 4m\omega^2 R = 4F_Z$

$\underline{\underline{F_Z' = 4F_Z}}$

e) Tourenzahl = Umdr. pro Zeit = Frequenz f

$f' = 2f$

$F_Z = m\omega^2 R = m(2\pi f)^2 R = m(2\pi)^2 f^2 R$

$F_Z' = m(2\pi f')^2 R = m(2\pi \cdot 2f)^2 R$

$= m(2\pi)^2 4f^2 R = 4 \cdot m(2\pi)^2 f^2 R = 4F_Z$

$\underline{\underline{F_Z' = 4F_Z}}$

(2) $R = 150\,m$; ges.: f, T, v

$a_Z = \omega^2 R = g = 9.81\,m/s^2$

$\omega = \sqrt{g/R}$

$f = \frac{1}{2\pi}\sqrt{g/R} \cong \underline{\underline{0.041\,Hz}}$

$T = \frac{1}{f} \cong \underline{\underline{24.57\,s}}$

$v = \omega R \cong \underline{\underline{38.36\,m/s \cong 138.1\,km/h}}$

(3) Im obersten Punkt ist $F_G = F_Z$:

$m\omega^2 R = mg \quad | :m :R$

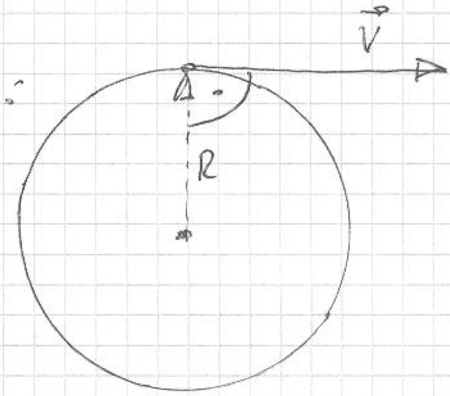
$\omega^2 = g/R$

$\omega = \sqrt{g/R}$

$f = \frac{1}{2\pi}\sqrt{g/R} \cong \underline{\underline{0.56\,Hz}}$

④ $R = 0.2\,m$, $f_H = 2.4$

Klotz fliegt tangential weg:



Reibkraft = Zentripetalkraft

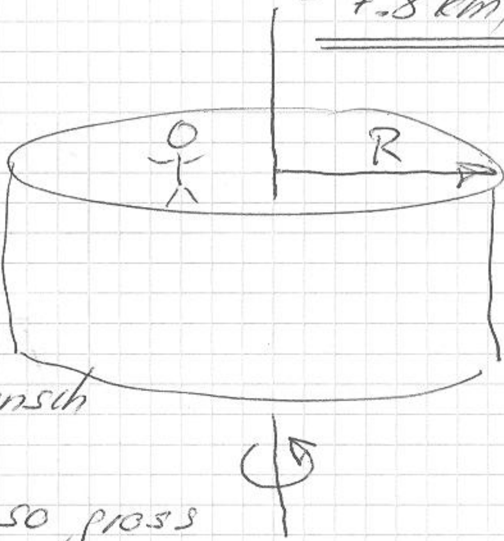
$$F_{Reib} = F_Z$$

$$f_H \cdot m \cdot g = \frac{mv^2}{R} \quad / : m \cdot R$$

$$R \cdot f_H \cdot g = v^2 \,; \quad v = \sqrt{R \cdot f_H \cdot g} \cong 2.17\,m/s$$

$$\cong \underline{\underline{7.8\,km/h}}$$

⑤ $R = 3\,m$, $f_H = 0.75$



$$F_{Reib} = f_H \cdot F_{Normal}$$

$$F_{Normal} = F_Z$$

$$F_{Reib} = mg \,; \quad m = \text{Masse Mensch}$$

die Reibkraft muss min. so gross wie die Gewichtskraft mg des Menschen sein:

$$F_G = m \cdot g = F_{Reib} = f_H \cdot F_{Normal} = f_H \cdot F_Z$$

$$mg = f_H \cdot m\omega^2 R$$

$$\omega^2 = \frac{g}{f_H \cdot R} = (2\pi f)^2 = \left(\frac{2\pi}{T}\right)^2$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{f_H R} \quad / \; 1/x$$

$$\left(\frac{T}{2\pi}\right)^2 = \frac{f_H \cdot R}{g}$$

$$T^2 = \frac{(2\pi)^2 f_H \cdot R}{g} \Rightarrow \underline{\underline{T \cong 3s}}$$

$$v \cong 6.24\,m/s \cong 22.6\,km/h$$

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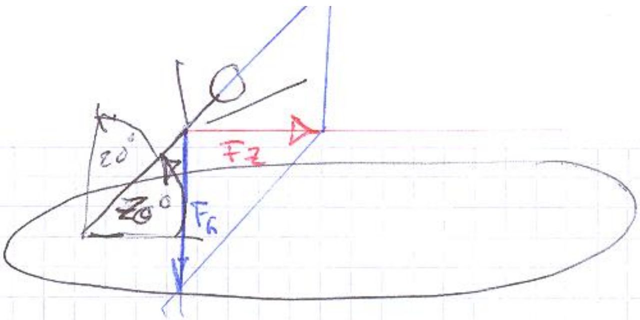


$$\tan\alpha = \frac{F_Z}{F_G}$$

$$\tan\alpha = \frac{m\omega^2 R}{mg} = \frac{\omega^2 R}{g}$$

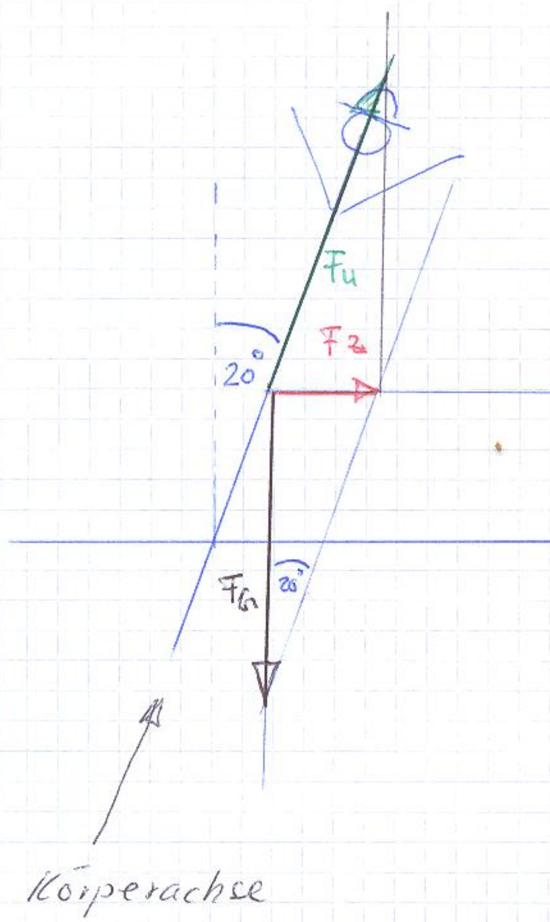
$$\omega^2 = \frac{(2\pi)^2}{T^2} = \frac{g\cdot\tan\alpha}{R}$$

$$\frac{T^2}{(2\pi)^2} = \frac{R}{g\cdot\tan\alpha}$$

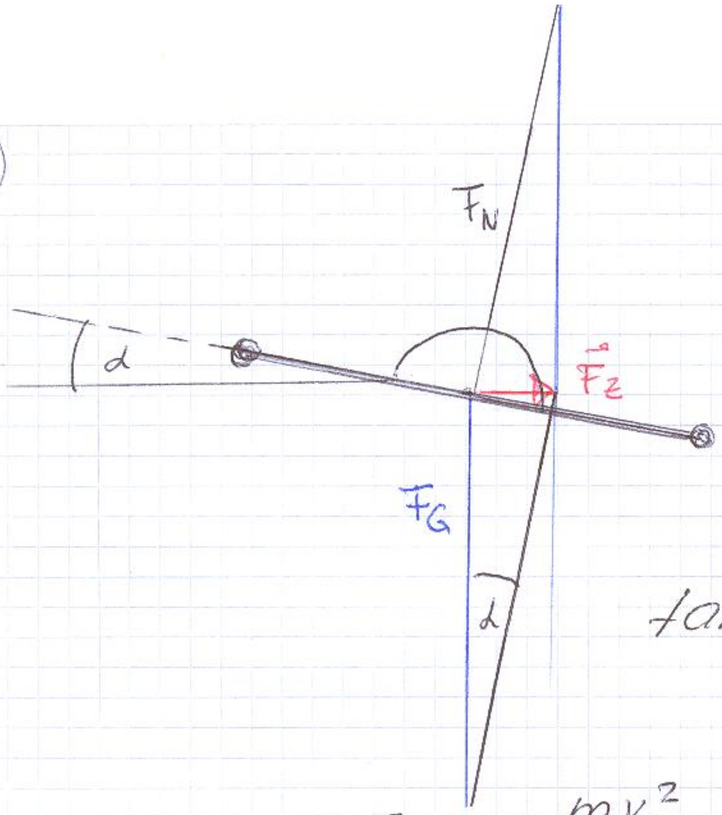
$$T = 2\pi\cdot\sqrt{R/g\cdot\tan\alpha}$$

$$= 2\pi\sqrt{\frac{7\,m}{9.81\,m/s^2\cdot\tan 20^\circ}}$$

$$\cong \underline{\underline{8.8\,s}}$$



(7)



α := Neigunswinkel des Flugzeugs

F_N: "Normalkraft", wirkt $\perp$ zu Flügel

$$\tan\alpha = \frac{F_Z}{F_G}$$

b) $$\tan\alpha = \frac{F_Z}{F_G} = \frac{\frac{mv^2}{R}}{mg} = \frac{v^2}{gR}$$

$$\Rightarrow \quad v^2 = g \cdot R \cdot \tan\alpha$$

$$\Rightarrow \quad v = \sqrt{g \cdot R \cdot \tan 80^\circ} \cong 235.9 \, m/s$$

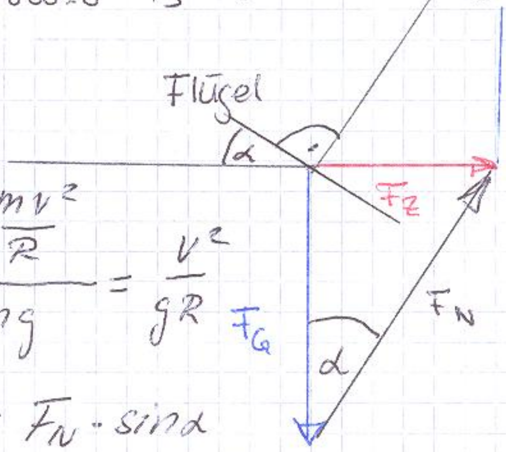
$$\cong \underline{\underline{849.1 \, \cancel{m/s} \, km/h}}$$

c) $v \longrightarrow \infty$ für $\alpha \longrightarrow 90^\circ$

(8) $a_{max} = 10g = 98.1 \, m/s^2$

$F_N = m \cdot a = 10mg$ = Normalkraft

$v = 2'400 km/h = 666.\bar{6} \, m/s$ (siehe Aufg. 7)



1.) $\tan\alpha = \frac{F_Z}{F_G} = \frac{\frac{mv^2}{R}}{mg} = \frac{v^2}{gR}$

2.) $\sin\alpha = \frac{F_Z}{F_N} \Rightarrow F_Z = F_N \cdot \sin\alpha$

F_Z aus 2.) in 1.) einsetzen:

$$\tan\alpha = \frac{F_N \cdot \sin\alpha}{F_G}$$

$$\tan\alpha = \frac{10mg \cdot \sin\alpha}{mg} = 10 \sin\alpha$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = 10 \cdot \sin\alpha \quad / : \sin\alpha$$

$$\frac{1}{\cos\alpha} = 10 \quad / \, 1/x$$

$$\cos\alpha = \frac{1}{10} \Rightarrow \alpha = \arccos\frac{1}{10} \cong 84.26°$$

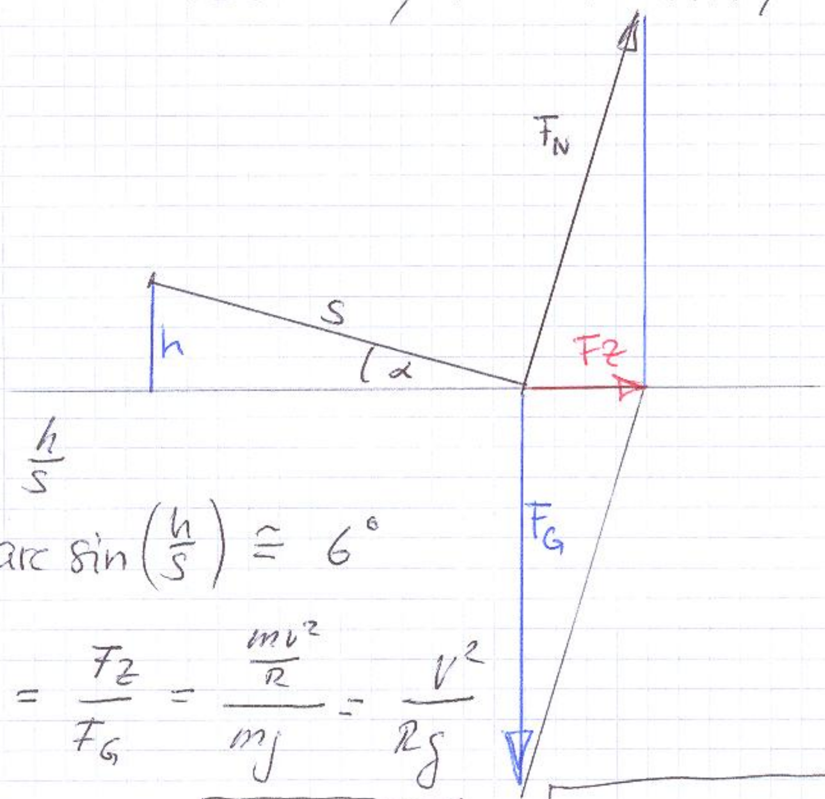
α einsetzen in $\tan\alpha = \frac{v^2}{gR} \Rightarrow R = \frac{v^2}{g \cdot \tan\alpha}$

$\cancel{v^2 = gR \cdot \tan\alpha}$

$\cancel{v = gR}$

$$R \cong 4'553.35 \, m$$

(9) $S = 1435\,mm,\ R = 150\,m,\ h = 15\,cm$



$\sin\alpha = \frac{h}{s}$

$\alpha = \arcsin\left(\frac{h}{s}\right) \cong 6°$

$\tan\alpha = \frac{F_Z}{F_G} = \frac{\frac{mv^2}{R}}{mg} = \frac{v^2}{Rg}$

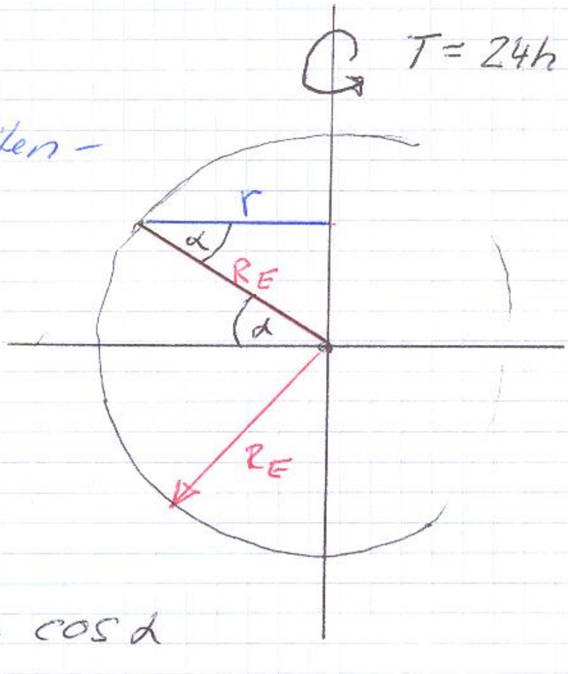
$\Rightarrow\ v = \sqrt{Rg\tan\alpha} = \sqrt{R\cdot g\cdot\tan\left(\arcsin\frac{h}{s}\right)}$

$\cong 12.44\,m/s \cong \underline{\underline{44.8\,km/h}}$

(10) α = Breitengrad

R_E = Erdradius

r = Radus auf Breiten-grad α



$\cos\alpha = \frac{r}{R_E}$

$r = R_E\cdot\cos\alpha$

$v = \omega\cdot r = \frac{2\pi}{T}\cdot r$

$v = \frac{2\pi}{24\cdot 3600\,s}\cdot R_E\cdot\cos\alpha$

$\alpha = 60°:\ v \cong 231.7\,m/s \cong 834\,km/h$

$\alpha = 45°:\ v \cong 327.6\,m/s \cong 1'179.4\,km/h$