G2e; Mo. 24.1.2011:

Vektorielle Kräfte

(1) $F_1 = 5N$, $\vec{F}_1 \parallel \vec{F}_2 = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$

$F_2 = \sqrt{15^2 + 20^2} = 25$; $\frac{\vec{F}_2}{F_2} = \frac{1}{25} \cdot \begin{pmatrix} 15 \\ 20 \end{pmatrix}$

$\vec{F}_1 = 5 \cdot \frac{1}{25} \cdot \begin{pmatrix} 15 \\ 20 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 15 \\ 20 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\underline{\underline{\vec{F}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} N}}$ Test: $\sqrt{3^2 + 4^2} = 5$

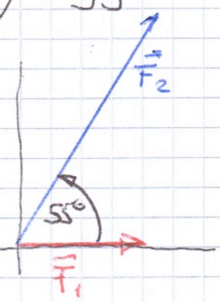
(2) $F_1 = 200N$, $F_2 = 300N$, $\alpha = \sphericalangle(\vec{F}_1, \vec{F}_2) = 55°$

$\vec{F}_1 = \begin{pmatrix} 200 \\ 0 \end{pmatrix}$, $\vec{F}_2 = 300 \begin{pmatrix} \cos 55° \\ \sin 55° \end{pmatrix}$

$\vec{F}_1 + \vec{F}_2 = \begin{pmatrix} 200 + 300 \cdot \cos 55° \\ 300 \cdot \sin 55° \end{pmatrix}$

$|\vec{F}_1 + \vec{F}_2| = \sqrt{(200 + 300 \cdot \cos 55°)^2 + (\sin 55° \cdot 300)^2}$

$\underline{\underline{\cong 445.9N}}$



(3) $F_\perp = F_G \cdot \cos\alpha = mg\cos\alpha = 19.47N$

$F_\parallel = F_G \cdot \sin\alpha = mg\sin\alpha = 2.39N$

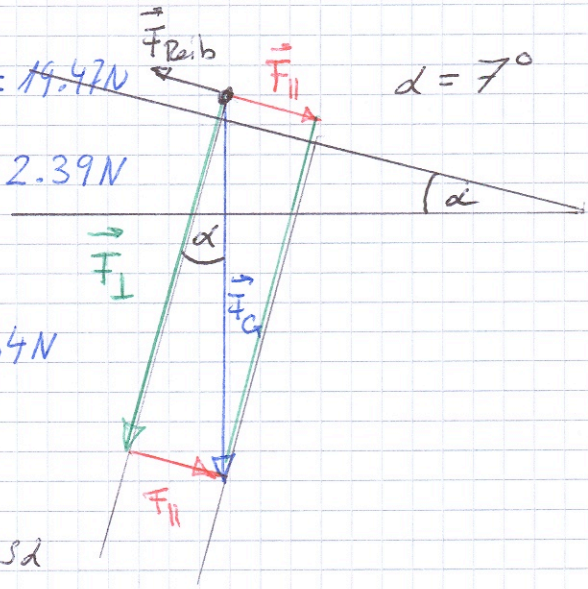
$F_{Reib} = f_H \cdot F_\perp$

$= f_H \cdot mg \cdot \cos\alpha = 2.34N$

$\vec{F}_{Res} = \vec{F}_\parallel - \vec{F}_{Reib}$

$= mg\sin\alpha - f_H \cdot mg \cdot \cos\alpha$

$= mg\underbrace{(\sin\alpha - f_H \cdot \cos\alpha)}_{(*)} = \underline{\underline{0.054N}}$



$F_\parallel > F_{Haftreibung} \Longrightarrow$ Würfel rutscht

b) $F_{Res} = mg(\sin\alpha - f_H \cdot \cos\alpha)$

$F_{Res} > 0 \Rightarrow$ rutscht

$F_{Res} \leq 0 \Rightarrow$ rutscht nicht

ob F_{Res} kleiner oder grösser Null ist hängt nur von klammer $(\sin\alpha - f_H \cdot \cos\alpha)$ ab, nicht von der Masse!

(4) $\vec{F}_1 = 5N \cdot \begin{pmatrix} \cos 160° \\ \sin 160° \end{pmatrix}$; $\vec{F}_2 = 3N \cdot \begin{pmatrix} \cos 60° \\ \sin 60° \end{pmatrix}$

$\vec{F}_1 + \vec{F}_2 = \begin{pmatrix} 5 \cdot \cos 160° + 3 \cdot \cos 60° \\ 5 \cdot \sin 160° + 3 \cdot \cos 60° \end{pmatrix} \cong \begin{pmatrix} -3.2 \\ 4.31 \end{pmatrix}$

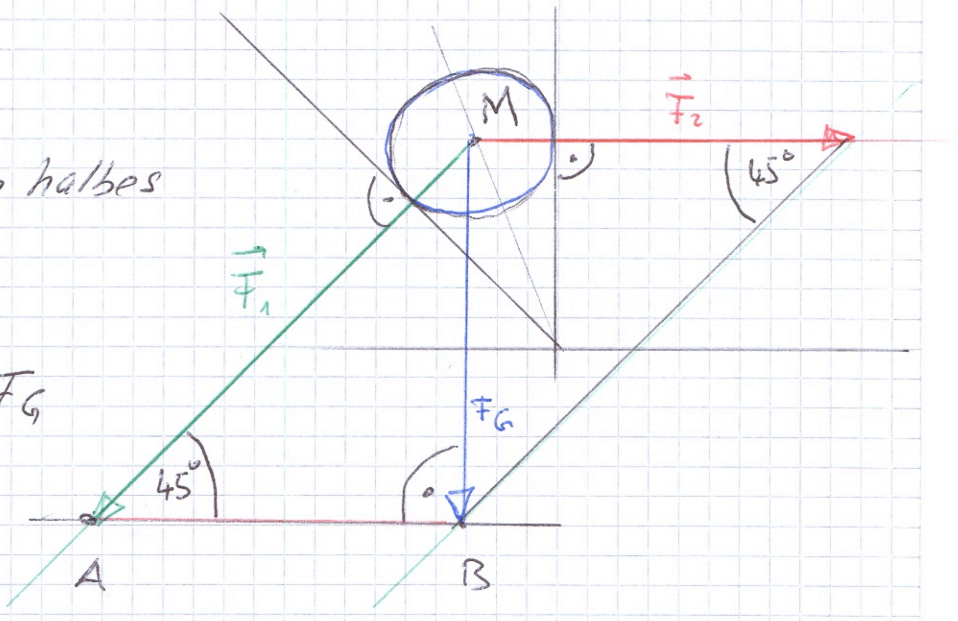
$|\vec{F}_1 + \vec{F}_2| = \sqrt{(+3.2)^2 + 4.31^2} \cong \underline{\underline{5.37N}}$

(5) $m = 20kg$

ΔABM ist ein halbes Quadrat:

$\Rightarrow F_2 = F_G$

$\Rightarrow F_1 = \sqrt{2}\, F_G$

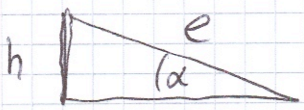


$F_1 = \sqrt{2} \cdot F_G = \sqrt{2} \cdot mg = \underline{\underline{277.47N}}$

$F_2 = F_G = mg = \underline{\underline{196.2N}}$

6 $m = 5\,kg$, $l = 5\,m$, $\alpha = 30°$, $v = 3.88\,m/s$

$h = l \cdot \sin\alpha$



Nach Aufg. 3 gilt:

$F_{\perp} = F_G \cdot \cos\alpha$

$F_{\parallel} = F_G \cdot \sin\alpha$

$$E_{pot} = E_{kin} + E_{Reibung}$$

$$mgh = \frac{1}{2}mv^2 + f_{gl} \cdot F_{\perp} \cdot l$$

$$mgl \cdot \sin\alpha = \frac{1}{2}mv^2 + f_{gl} \cdot mgl\cos\alpha \quad | : m$$

$$gl \cdot \sin\alpha = \frac{1}{2}v^2 + f_{gl} \cdot gl\cos\alpha \quad | - \frac{1}{2}v^2$$

$$gl \cdot \sin\alpha - \frac{1}{2}v^2 = f_{gl} \cdot gl\cos\alpha \quad | : (gl\cos\alpha)$$

$$\frac{gl \cdot \sin\alpha - \frac{1}{2}v^2}{gl\cos\alpha} = f_{gl} = \underline{\underline{0.4}}$$

$E_{pot} = 122.625\,J$

$E_{kin} = 37.636\,J$ −

$E_{Reib} = 84.989\,J$