31C, 2.12.2008
Statik und Energieerhaltung

(1) Summe der Drehmomente = 0

$$\sum_i \vec{M}_i = 0$$

Unterster Balken: $0.03m \cdot 2kg \cdot g = 0.04m \cdot m_1 \cdot g$

$$3 \cdot 2kg = 4 \cdot m_1$$

$$\frac{3}{4} \cdot 2kg = m_1 = 1.5kg$$

Gesamte Masse unterster Balken:

$$m_u = 2kg + 1.5kg = 3.5kg$$

Mittlerer Balken: $m_2 \cdot 2 = 4 \cdot 3.5kg$

$$m_2 = 7kg$$

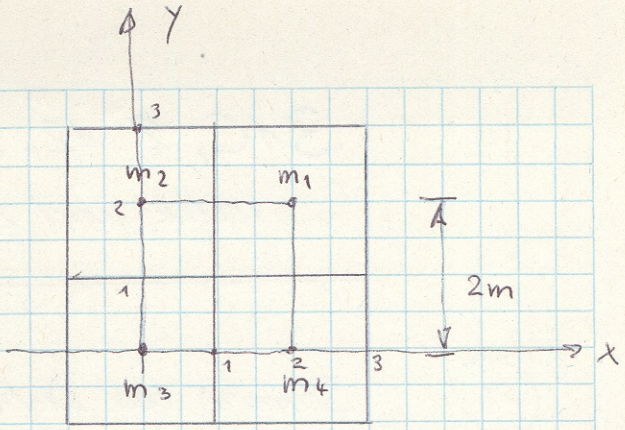
Gesamte Masse mittlerer Balken:

$$m_m = m_2 + m_u = 7kg + 3.5kg = 10.5kg$$

oberster Balken: $10.5kg \cdot 2 = m_3 \cdot 6$

$$\frac{21}{6}kg = m_3 = 3.5kg$$

(2)



$$\vec{r}_S = \frac{1}{M}\sum_i \vec{r}_i m_i$$

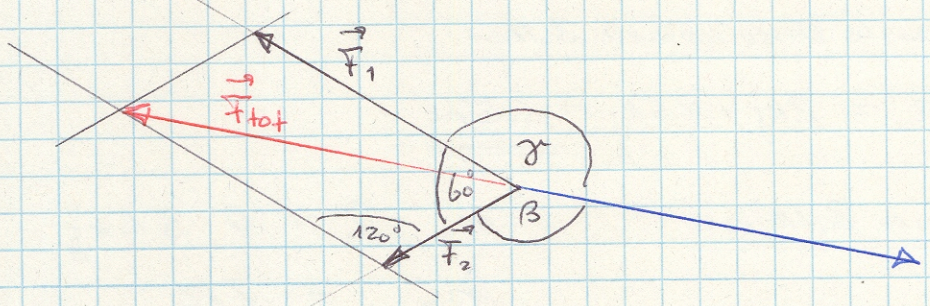
$$= \frac{1}{30+6+40+50}\left(30\begin{pmatrix}2\\2\end{pmatrix} + 6\begin{pmatrix}0\\2\end{pmatrix} + 40\begin{pmatrix}0\\0\end{pmatrix} + 50\begin{pmatrix}2\\0\end{pmatrix}\right)$$

$$\vec{r}_1 = \begin{pmatrix}2\\2\end{pmatrix},\ \vec{r}_2 = \begin{pmatrix}0\\2\end{pmatrix},\ \vec{r}_3 = \begin{pmatrix}0\\0\end{pmatrix},\ \vec{r}_4 = \begin{pmatrix}2\\0\end{pmatrix}$$

$$= \frac{1}{126}\left(\begin{pmatrix}60\\60\end{pmatrix} + \begin{pmatrix}6\\12\end{pmatrix} + \begin{pmatrix}100\\0\end{pmatrix}\right)$$

$$= \frac{1}{126}\begin{pmatrix}160\\72\end{pmatrix} = \begin{pmatrix}160/126\\72/126\end{pmatrix} = \begin{pmatrix}80/63\\4/7\end{pmatrix} \approx \underline{\underline{\begin{pmatrix}1.27\\0.57\end{pmatrix}}}$$

(3)

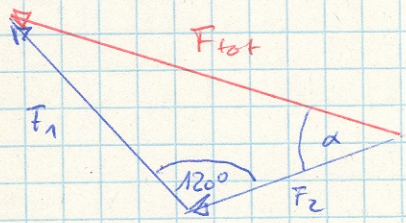


Cosinus-Satz:

$$F_{tot}^2 = F_1^2 + F_2^2 - 2F_1F_2\cos 120°$$

$$F_{tot} = \sqrt{200^2 + 100^2 - 2\cdot 200\cdot 100\cdot \cos 120°}$$

$$\approx \underline{\underline{264.58\,N}}$$



Sinus-Satz:

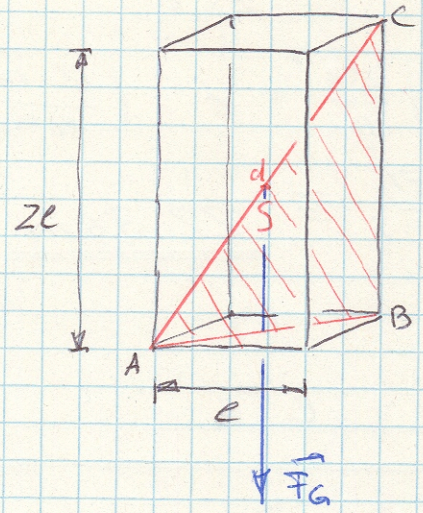
$$\frac{\sin 120°}{F_{tot}} = \frac{\sin\alpha}{F_1}$$

$$\frac{F_1}{F_{tot}}\cdot\sin 120° = \sin\alpha$$

$$\alpha = \arcsin\left(\frac{F_1}{F_{tot}}\cdot\sin 120°\right) \approx \underline{\underline{40.9°}}$$

$$\beta = 180° - \alpha = \underline{\underline{139.1°}},\quad \gamma = 36° + 60° - \beta = \underline{\underline{160.9°}}$$

(4)



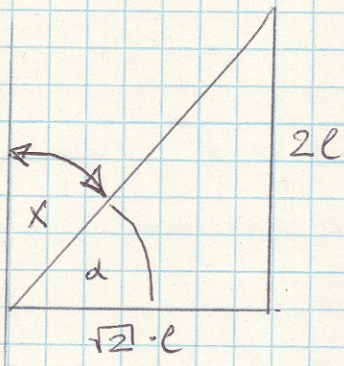
Der Schwerpunkt (S) liegt in der Mitte der Raumdiagonalen (d)

$\vec{F}_G$ zeigt bezüglich Erde immer senkrecht nach unten. Quader kann gekippt werden, bis die Linie von $\vec{F}_G$ auf d liegt, dass heisst, bis d senkrecht zum Boden steht:

α: $\tan\alpha = \frac{2\ell}{\sqrt{2}\,\ell} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\alpha \approx 54.74°$

$x = 90° - \alpha \approx \underline{35.26°}$



(5) $m = 10g$, $k = 500 Nm^{-1}$, $x_1 = 20cm$, $x_2 = 30cm$

$\Delta E = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}\cdot 500\,(0.09 - 0.04)$

$= \frac{1}{2}\cdot 500 \cdot 0.05 = \underline{12.5 \text{ Joules}}$

a) $E_{pot} = E_{Feder}$: $m \cdot g \cdot h = 12.5J$

$h = 12.5J / mg = 125$ Meter

b) $E_{kin} = \frac{1}{2}mv^2 = E_{Feder}$: $\frac{1}{2}mv^2 = 12.5J$

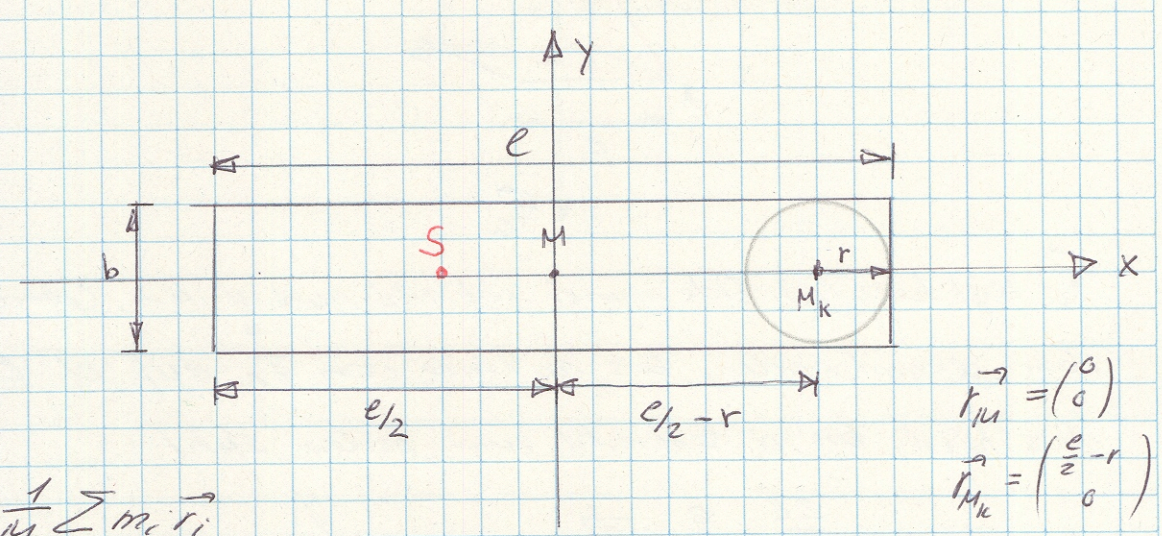
$v = \sqrt{\frac{25J}{0.01kg}} = 50\,\frac{m}{s}$

Flugzeit 1 Meter Höhe: $s = \frac{1}{2} g t^2$

$$\sqrt{\frac{2s}{g}} = t = \sqrt{\frac{2 \cdot 1m}{10\, m/s^2}} = \frac{1}{\sqrt{5}} s = \frac{\sqrt{5}}{5} s$$

$$s = v \cdot t = 50 \frac{m}{s} \cdot \frac{\sqrt{5}}{5} s \approx \underline{22.36m}$$

(6)



$$\vec{r}_M = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \vec{r}_{M_K} = \begin{pmatrix} \frac{e}{2} - r \\ 0 \end{pmatrix}$$

$$\vec{r}_S = \frac{1}{M} \sum m_i \vec{r}_i$$

Massen: $m = \rho \cdot V$ (Dichte · Volumen)

Brett ohne Loch: $\rho \cdot e \cdot b \cdot d = m_B$ (d = Dicke)

Loch: $\rho \cdot \pi \cdot r^2 d = m_L$

M = Gesamtmasse = $m_{Brett} - m_{Loch}$

$$\vec{r}_S = \frac{1}{m_B - m_L} \left(m_B \begin{pmatrix} 0 \\ 0 \end{pmatrix} - m_L \begin{pmatrix} e/2 - r \\ 0 \end{pmatrix} \right)$$

$$= \frac{1}{\rho e b d - \rho \pi r^2 d} \left(- \rho \pi r^2 d \begin{pmatrix} \frac{e}{2} - r \\ 0 \end{pmatrix} \right)$$

$$= \frac{1}{\rho d (eb - \pi r^2)} \begin{pmatrix} \quad \\ 0 \end{pmatrix} \Rightarrow \text{y-koord.} = 0$$

x-koordinate: $S_x = \dfrac{-\rho \pi r^2 d \left(\frac{e}{2} - r \right)}{\rho d (eb - \pi r^2)} = \dfrac{-\pi r^2 \left(\frac{e}{2} - r \right)}{eb - \pi r^2}$

$$= - \frac{\left(\frac{e}{2} - r \right)}{\frac{eb}{\pi r^2} - 1} = - \frac{1.5 - 0.2}{\frac{3 \cdot 0.6}{\pi \, 0.2^2} - 1} \approx -9.8 cm$$