Musterlösungen Thermodynamik

Skript S. 11f; Vermischte Aufgaben

(1) $T = 0°C$, $p = 1\,bar$;

Nehme 1 mol Gas und berechne Volumen. $1\,mol = 6.02 \cdot 10^{23}$ Teilchen: rechne Volumen pro Teilchen. Betrachte Volumen pro Teilchen als Würfel $\Rightarrow \sqrt[3]{V}$ = mittlerer Abstand

$$p \cdot V = nRT \iff V = \frac{nRT}{p} \quad ; \; n = 1$$

$$(R = k_B \cdot N_A) \qquad V_{\text{pro Teilch.}} = \frac{V}{N_A} = \frac{nRT}{pN_A} = \frac{k_B \cdot T}{p}$$

$$\text{Mittlerer Abstand} = \sqrt[3]{V_{\text{pro Teilchen}}} = \sqrt[3]{\frac{k_B \cdot T}{p}} \cong \underline{\underline{3.4\,nm}}$$

Vergleich: O-Atom ~ 60 pm, O_2-Molekül ~ 300 pm

(2) $T_1 = 20°C$, $p = 1\,bar$, $T_2 = 5°C$, $p_2 = 0.8\,bar$

Ballon: Innendruck = Aussendruck; $n = const.$

$$pV = nRT \iff V = \frac{nRT}{p}$$

$$\left.\begin{aligned} V_1 &= \frac{nRT_1}{p_1} \\ V_2 &= \frac{nRT_2}{p_2} \end{aligned}\right\} \Rightarrow \frac{V_1}{V_2} = \frac{\frac{nRT_1}{p_1}}{\frac{nRT_2}{p_2}} = \underline{\underline{\frac{T_1 p_2}{T_2 p_1} = 0.843}}$$

0.843 l Gas am Boden dehnt sich auf 1 l aus in 2500 m

(3) $20\,l,\ T = 20°C,\ p = 100\,bar = 10^7\,Pa,\ M_{He} = 4$

$pV = nRT$; gesucht: n (Anzahl Mole)

$$n = \frac{pV}{RT}$$

$m = n \cdot$ molare Masse $= n \cdot (4g/mol)$

$$m = \frac{pV}{RT} \cdot 0.004\,\frac{kg}{mol} \cong 0.328\,kg = \underline{\underline{328g}}$$

Ballon

$V = 5l,\ p = 1.2\,bar,\ T = 20°C$

$$n = \frac{pV}{RT},\quad m = \frac{pV}{RT} \cdot \left(0.004\,\frac{kg}{mol}\right) \cong 0.985g$$

↪ $\underline{\underline{333 \text{ Ballone}}}$

(4) $p = 20\,bar = 2 \cdot 10^6\,Pa$

$V_2 = 3/4 \cdot V_1$; $T_1 = T_2$

$$\frac{p_1 V_1}{p_2 V_2} = \frac{nRT_1}{nRT_2} = 1 \iff p_1 V_1 = p_2 V_2 \quad \text{(Boyle-Mariotte)}$$

$$V_2 = \frac{3}{4} V_1 \implies p_1 V_1 = p_2 \cdot \frac{3}{4} V_1$$

$$p_2 = \frac{4}{3} p_1 = \frac{4}{3} \cdot 20\,bar \cong \underline{\underline{26.\bar{6}\,bar}}$$

(5) $$\frac{p_1 V_1}{p_2 V_2} = \frac{nRT_1}{nRT_2} = \frac{T_1}{T_2} \qquad V_1 = V_2$$

$$\implies \frac{p_1}{p_2} = \frac{T_1}{T_2} \implies \underline{\underline{p_2 = p_1 \frac{T_2}{T_1} = 120\,bar \cdot \frac{293.15}{328.15} \cong 134.3\,bar}}$$

(6) $L_{0°} = 4cm$; $L_{100°} = 24cm$; $T_C(L) = \frac{L - L_{0°}}{L_{100°} - L_{0°}} \cdot 100°C$

a) $T_C(L) = 22°C = \frac{L - 0.04m}{0.24m - 0.04m} \cdot 100°C \quad | : 100°C$

$$\frac{22}{100} = \frac{L - 0.04m}{0.2m}$$

$$\frac{11}{55} \cdot 0.2m + 0.04m = L = 0.084m = \underline{\underline{8.4cm}}$$

b) $T_C(0.254m) = \frac{0.254m - 0.04m}{0.24m - 0.04m} \cdot 100°C = \underline{\underline{107°C}}$

(7) $V = const.$, $p_1 = 0.4 atm$ bei $0°C$

$p_2 = 0.546 atm$ bei $100°C$

Für Gasthermometer gilt (analog zu $T_C(L) = \frac{L - L_0}{L_{100} - L_0} \cdot 100°C$)

$$\boxed{t_{Celsius}(p) = \frac{p - p_0}{p_{100°} - p_{0°}}} \cdot 100°C$$

Herleitung:

$$\left.\begin{array}{l} p_0 V = nRT_0 \\ p_{100} V = nRT_{100} \end{array}\right\} (p_{100} - p_0)V = nR\underbrace{(T_{100°} - T_{0°})}_{= 100°C}$$

$$\underline{\underline{(p_{100} - p_0)V = nR \cdot 100°C}} \quad (I)$$

~~p_{100}~~ p

~~$pV = nRT$~~

~~$p_{100}V = nRT_{100}$~~ } ~~$(p - p_{100})V =$~~

$$\left.\begin{array}{l} pV = nRT \\ p_0 V = nRT_{0°} \end{array}\right\} (p - p_0)V = nRT - \underbrace{nRT_{0°}}_{= 0}$$

$$\underline{\underline{(p - p_0)V = nRT}} \quad (II)$$

I. : II. :

$$\frac{(p_{100}-p_0)V}{(p-p_0)V} = \frac{nR\cdot 100\,^\circ C}{nRT}$$

$$T(p) = \frac{p-p_0}{p_{100}-p_0}\cdot 100\,^\circ C$$

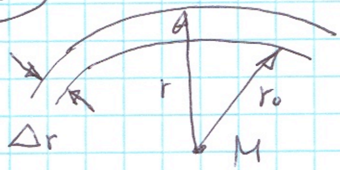
a) $T(0.1\,atm) = ?$

$$T = \frac{0.1\,atm - 0.4\,atm}{0.546\,atm - 0.4\,atm}\cdot 100\,^\circ C = \underline{\underline{-205.5\,^\circ C}}$$

b) $$T(p) = 444.6\,^\circ C = \frac{p - 0.4\,atm}{0.546\,atm - 0.4\,atm}\cdot 100\,^\circ C$$

$$\frac{444.6\,^\circ C \cdot 0.146\,atm}{100\,^\circ C} + 0.4\,atm \cong \underline{\underline{1.05\,atm}}$$

(8)



$\Delta r = r - r_0\,;\quad U = 2\pi r\,,\quad r = \frac{U}{2\pi}$

$r_{Erde} = 6.37\cdot 10^6\,m = r_0$

$\alpha_{Stahl} = 11\cdot 10^{-6}\,K^{-1}$

$L' = L\,(1+\alpha\cdot\Delta T)$

$$\left.\begin{aligned} U_0 &= 2\pi r_0 \\ U &= 2\pi r \end{aligned}\right\}\quad \Delta h = r - r_0 = \frac{U}{2\pi} - \frac{U_0}{2\pi} = \frac{U-U_0}{2\pi} = \frac{\Delta L}{2\pi}$$

$$\Delta h = \frac{\Delta L}{2\pi} = \frac{L\cdot\alpha\cdot\Delta T}{2\pi} \cong$$

$$= \frac{U_0}{2\pi}\cdot\alpha\cdot\Delta T = r_0\cdot\alpha\cdot\Delta T$$

$$= \underline{\underline{2'102.1\ \text{Meter}}}$$

(9) Standard-Bed.: $p = 1atm = 101'325\ Pa$

$T = 0°C$

$$pV = nRT \Leftrightarrow V = \frac{nRT}{p} = 2.2414 \cdot 10^{-2}\ m^3$$

$$= \underline{\underline{22.414\ Liter}}$$

(10) $n = 1mol,\ p = 1atm,\ V = 10l$

a) $pV = nRT \Rightarrow T = \frac{pV}{nR} = \frac{10^5 Pa \cdot 0.01 m^3}{n \cdot R} = \underline{\underline{121.87 K}}$

b) $p = 1atm,\ V = 20l$

$\underbrace{pV = nRT \quad ;\ p = const}$

$\Rightarrow$ T proportional zu V $\Rightarrow \underline{\underline{T = 243.73 K}}$

"doppeltes Volumen $\Rightarrow$ doppelte Temperatur"

c) $V = 20l,\ T = 350K$

$pV = nRT \Rightarrow p = \frac{nRT}{V} = \underline{\underline{1.46\ bar}}$

(11) $\left.\begin{array}{l} p_1 V_1 = nRT_1 \\ p_2 V_2 = nRT_2 \end{array}\right\}$ $p = const.$

$$\frac{p_1 V_1}{p_2 V_2} = \frac{nRT_1}{nRT_2}$$

$$\Downarrow$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \Leftrightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1} \Leftrightarrow V_2 = V_1 \frac{T_2}{T_1}$$

$$= V_1 \frac{373.15}{323.15} = \underline{\underline{1.15}}$$

(12) $p = 10^{-8}$ Torr $1\,Torr = \frac{1}{760}\,atm$

$T = 300K$ $1\,atm = 101325\,Pa$

$$p \cdot V = nRT = N \cdot k_B \cdot T$$

$$\frac{p}{k_B T} = \frac{N}{V} \; ; \quad \frac{N}{V} = \text{"Teilchen pro Volumen"}$$

$$\frac{p}{k_B T} = \frac{10^{-8} \cdot \frac{1}{760} \cdot 101325\,Pa}{k_B \cdot 300K} = 3.22 \cdot 10^{14}\,m^{-3}$$

$$1m^3 = 10^6 cm^3 \longrightarrow \underline{\underline{3.22 \cdot 10^8\,cm^{-3}}}$$

Technisch möglich : ~ 10'000 Teilchen pro cm^3 (UHV)

Weltraum : ~ 1 " " "

(13) $p = 1atm$, $T = 300K$, $V = 6m \times 5m \times 3m = 90m^3$

$$pV = nRT \Leftrightarrow n_{300K} = \frac{pV}{RT} \cong \underline{\underline{3'656\,mol}}$$

b) $T \uparrow 5K$: Bei höherer Temp. braucht es bei gleichem Volumen weniger Teilchen, um den gleichen Druck zu erzeugen.

$$n_{305K} = \frac{p \cdot V}{RT} = \frac{101325Pa \cdot 90m^3}{R \cdot 305K}$$

$$\cong \underline{\underline{3'596\,mol}}$$

$$n_{300K} - n_{305K} \cong \underline{\underline{60\,mol}}$$

Andere Betrachtung:

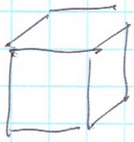
$T \uparrow$; vergrössere Volumen, damit $p = const.$ bleibt

$$V' = \frac{nRT}{p} = \frac{3656 \cdot R \cdot 305}{101325Pa} \cong 91.5\,m^3$$

es entweicht $\frac{1.5m^3}{91.5m^3} \cdot 3656\,mol \cong \underline{\underline{60\,mol}}$

Verm. Aufgaben, Skript S. 11f

(14)



$P_1 = 1atm = 101'325\,Pa$

$T_1 = 300K \;/\; T_2 = 400K$

Gesucht: p_2 ; ($n = const.$, $V = const.$)

$$\left.\begin{array}{l} p_1 V = nRT_1 \\ p_2 V = nRT_2 \end{array}\right\} \Rightarrow \frac{p_1 V}{p_2 V} = \frac{nRT_1}{nRT_2} \Leftrightarrow \frac{p_1}{p_2} = \frac{T_1}{T_2}$$

$$p_2 = p_1 \frac{T_2}{T_1} = 101325\,Pa \cdot \frac{400K}{300K} \Leftrightarrow \frac{p_1 T_2}{T_1} = p_2$$

$\cong 135'100\,bar$

Kraft auf Wand: $p = \frac{F}{A} \Leftrightarrow F = p \cdot A = 5404\,N$

aber: von aussen wirkt normaler Luftdruck entgegen

$$F = 101'325\,Pa \cdot (0.2m)^2 = 4053\,N$$

$$\underline{\underline{\Delta F = 1351\,N}}$$

(15) $V = 60\,l$, $T_1 = 10°C$, $T_2 = 25°C$

$\gamma_{Benzin} = 0.9 \cdot 10^{-3}\,K^{-1}$; $\alpha_{Stahl} = 11 \cdot 10^{-6}\,K^{-1}$

Tank: $V' = V(1 + \gamma_{Stahl} \cdot \Delta T)$

$$= V(1 + 3\alpha_{Stahl} \cdot \Delta T) = 60.0297\,l$$

Benzin: $V' = V(1 + \gamma_{Benzin}\,\Delta T)$

$$= 60l(1 + 0.9 \cdot 10^{-3} \cdot 15°C) = 60.81\,l$$

Differenz: $\underline{\underline{0.7803\,l}}$

(16) $n = 1\,mol$

I.) $p_1 = 2\,atm,\ T_1 = 300\,K,\ V$

II.) $p_2 = 1\,atm,\ V',\ T_2 = ?$

III.) $p_3 = 2.5\,atm,\ V,\ T_3 = ?$

Zustand II.) ist irrelevant; es spielt keine Rolle, über welche Zwischenzustände Zustand III.) erreicht wird. Deshalb heisst $pV = nRT$ ZUSTANDSGLEICHUNG.

$$\left.\begin{array}{l} p_1 V = nRT_1 \\ p_3 V = nRT_3 \end{array}\right\} \Rightarrow \frac{p_1}{p_3} = \frac{T_1}{T_3} \Leftrightarrow T_3 = T_1 \cdot \frac{p_3}{p_1}$$

$$= 375\,K$$

(17) $h_1 = 40\,m,\ T_1 = 5°C,\ V_1 = 15\,cm^3$

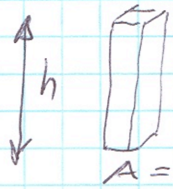
$h_2 = 0\,m \quad T_2 = 25°C,\ V_2 = ?$

Druck an Wasseroberfläche: $1\,atm = 101'325\,atm = p_1$

Druck in Tiefe h:

$$p = \frac{F}{A} = \frac{m \cdot g}{A} = \frac{\rho \cdot V \cdot g}{A}$$

$$= \frac{\rho \cdot A \cdot h \cdot g}{A} = \rho g h$$



$$\Rightarrow p_1 = p_2 + \rho g h \qquad \rho = \rho_{Wasser} \approx 10^3\,kg\,m^{-3}$$

Es ändern sich Druck und Volumen:

$$\left.\begin{array}{l} p_1 V_1 = nRT_1 \\ p_2 V_2 = nRT_2 \end{array}\right\} \Rightarrow \frac{p_1 V_1}{p_2 V_2} = \frac{T_1}{T_2} \Leftrightarrow \frac{p_2 V_2}{p_1 V_1} = \frac{T_2}{T_1}$$

$$\Leftrightarrow V_2 = \frac{p_1 V_1 T_2}{p_2 T_1} = \quad (101'325 + 10^3\, 9.81 \cdot 40\,Pa) \cdot 278\,K$$

$$= \frac{(101'325 + 10^3 \cdot 9.81 \cdot 40)\,Pa \cdot 15\,cm^3 \cdot 298\,K}{101'325\,Pa \cdot 278\,K} \cong 78.35\,cm^3$$